

UNIVERSITY OF CAMBRIDGE

FACULTY OF EDUCATION

EDUCATION TRIPOS PART 1

2008-2009

**MATHEMATICS EDUCATION**

*Tutors: Peter Huckstep and Tim Rowland*



## PART 1 MATHEMATICAL REASONING AND MATHEMATICS EDUCATION

The aim of the mathematics education component of the course is to introduce you to some of the major issues in mathematics education; in particular for you to engage with some literature in the discipline.

The syllabus is effectively defined by the prescribed set of readings.

### TEACHING METHODS:

1. The list of readings, in 8 'Blocks', is given in this handbook (p. 10).
2. A week-by-week pathway through the readings will be recommended. This timetable essentially defines the programme for class seminars and small-group supervisions.
3. The programme for class seminars in Michaelmas and Lent 2008-09 is given below.

You will be expected to have read the relevant set of readings before each seminar.

Michaelmas: Week 7	Peter	Block 1
Week 8	Tim	Block 2
Lent: Week 1	Peter	Block 3
Week 2	Tim	Block 4
Week 3	Peter	Block 5 [and essay guidance]
Week 4	Peter	Block 6
Week 5	Tim	Block 7
Week 6	Tim	Block 8

You will then be ready to work on the course assignment.

4. Supervision. Groups of 2 or 3 will be assigned a supervisor and arrange supervision times with them. We propose to use the time available as follows:
  - One 1-hour group supervision [Lent week 6]. Focus: reviewing the supervision essay (see 5. below).
  - Lent/Easter terms: Two 30-minute individual supervisions [see Timeline later]. Focus: the assessed essay. You must complete the essay title proposal form (at the back of this booklet) and bring it to the first of these supervisions.
5. Supervision Essay. Students will write one 1200-1500 word non-assessed supervision essay as follows:

Choose *one* of the six issues in mathematics education listed below and discuss how two or more of the blocks in the readings offer insights and/or perspectives on it.

1. Influences on the mathematics curriculum.
2. Approaches to arithmetic and calculation.
3. Aims and purposes of mathematics education.
4. Understanding in mathematics.
5. Radical/alternative approaches to mathematics teaching.

The essay should be handed in to your supervisor by the Monday of Lent Week 5. Formative feedback will be given in the group supervisions in the following week.

NOTE: In choosing the issue for this essay, bear in mind that you may not make the *same* issue the principal focus of your course assignment essay (although the focus of *that* essay is not restricted to the six listed above).

The purpose of the supervision essay is to develop your thinking and your writing style. For *this* essay, you are not expected to identify related literature in addition to that in the set of course readings. The essay should be word-processed. We expect you to read and observe the guidance on writing given later in this booklet, using the recommended form of citation and reference list..

## **6. Cambridge Mathematics Education Colloquia**

Two research seminars in mathematics education are held each term in Cambridge. Both are on Mondays 5.00-6.30 Further details of dates and speakers etc will be made available. You are warmly welcomed to the colloquia – and strongly encouraged to attend..

## **COURSE ASSESSMENT**

Students will write and submit one extended essay, on a topic negotiated and agreed with their supervisor. The topic must be related to one or more of the major themes of the course. The essay should be word-processed, and the pages numbered. The length is 4000 words ( $\pm 10\%$ ) for the Part 1 Exam.

We expect that your essay will reflect some selected reading beyond those in the set list. Your supervisor will help you to identify appropriate additional reading.

You will probably prepare and write the essay in the last two weeks of Lent and in the following vacation. Your individual supervisions are normally timed for Lent week 6/7 and the beginning of the week when you return to Cambridge for Easter term. You must complete the first part of the Essay Proposal Form at the back of this booklet and take it to the *first* of these individual supervisions.

At the second individual supervision you will get formative feedback on a draft of your assignment essay. It is therefore important (a) that you send the essay to your supervisor to read in advance of the supervision (b) that the work you give your supervisor is your best effort at the essay i.e with all sections, citations and references complete and properly formatted. It should *not* just be a plan, or notes, or ‘work in progress’, otherwise your supervisor will not be able to help you as s/he would wish to.

**Submit the essay on the first Friday of Easter term, in the Undergraduate Office.**

## **PLAGIARISM**

One of the purposes of proper citation is to avoid the serious offence of plagiarism, which is the presentation of someone else's work (published or unpublished) as though it were your own. This includes internet material. The consequences of plagiarism in work submitted for course assessment are very grave indeed.

## **JOURNALS**

Two journals are published by the Association of Teachers of Mathematics (ATM) *Mathematics Teaching* and *Micromath*. The Mathematical Association (MA) publishes *Mathematics in School* and *Equals*. The American association NCTM publishes *The Arithmetic Teacher*. There are several mathematics education research journals, including: *Educational Studies in Mathematics; For the Learning of Mathematics; Journal of Research in Mathematics Education*. The audience for these journals is primarily researchers, and they have a more international flavour.

SUMMARY AND TIMELINE 2008-09

DATE	WHAT YOU NEED TO DO
Every week	Read and annotate the set readings to prepare for weekly seminars
Lent week 4	Write your supervision essay.
Lent week 5	Monday: hand in the supervision essay to your supervisor
Lent week 6	Group meeting with your supervisor for feedback on the essay
Lent week 6/7	Take the Proposal Form at the back of this booklet to your first 30 minute individual supervision. Purposes (a) to agree the title (b) to consider broad structure (c) to identify some additional readings.
Lent weeks 6-8	Locate and borrow/copy additional readings identified at the supervision. Make a start on your course assignment essay.
Easter vacation	Complete the draft of your course assignment essay
Before 20 April 2009	Send your supervisor the draft of your essay
22-22 April	Second 30 minute individual supervision. Purposes: to discuss the draft.
24 April 2009	Complete essay and submit to the Undergraduate Office.



## WRITING ESSAYS: SOME GUIDELINES

The purpose of the supervised essay is formative; it is to assess your ability to engage in some chosen issues from the literature studied so far, enabling you to proceed to the assessed piece. It is also intended to help you to develop as a writer (a) of prose English (b) of academic ('scholarly') essays. The best way to develop is through practice and feedback. The guidance given below is of a general nature, and applies to the supervision essay and the assessed essay. Please read it carefully.

### Straightforward matters

There are a few common errors that frequently appear. With a little diligence, you can avoid all of them, using the following checklist.

1. Is your text an appropriate size (not less than 12 point font) and are the lines well-spaced?
2. Have you carefully followed the Harvard system of referencing? It is good to get into the habit of using the system *exactly*, not just near enough. This is one area where it is better *not* to be imaginative, creative, or to show initiative! Just do what it says. Take care with commas, full stops, inverted commas, italics. See below for further guidance on citation and referencing. If in doubt, ask your supervisor for clarification.
3. Are all your references complete? For books, it is preferable to give the *place* of publication (e.g. London: Routledge). For journal articles, it is *essential* to give page numbers. (It is also good practice to do this for chapters in books).
4. Is every author who is mentioned ('cited') in your work linked to an entry in the list of references at the end? This is essential, so that the interested reader can follow it up. In the case of 'secondary references', where, for example, one author has cited another within his/her text, you need only enter the author, date, source and publisher of the 'secondary source' and mention that the primary reference is quoted in that work.

For example, "Feiman-Nemser and Buchmann (quoted in Wilson, Shulman and Richerd, 1987, p.115) characterise 'pedagogical thinking' as "strategic, imaginative, and grounded in knowledge of self, children and subject matter'." In this case, Wilson, Shulman and Richerd, 1987, is the secondary source. By citing the secondary, you acknowledge that you have not read the primary source.

5. Does every entry in the list of references correspond to some citation in your work? The list of references should *not* be used to display all of the books that you have consulted in preparation for the work (which might be called a 'bibliography'), *only* those to which you explicitly refer in the body of the essay.
6. Check carefully for common errors that your spell-check will not detect. For example, 'it's' is a contraction of 'it is', whereas 'its' means 'belonging to it'; 'practice' is a noun, 'practise' is a verb. Use a dictionary to check that you know the difference between 'effect' and 'affect'.
7. Check the spelling of your authors – it should be Bruner not 'Burner', Skemp not 'Skamp' (and it's always nice to spell correctly the name of anyone who might read your essay!). Check their gender if necessary, so that correct pronouns (he/she) can be used in your essay.

### Citation and Referencing

#### Citation in the text – for example

(Morgan, 1998) *or* "as Morgan (1998) argues". If you're giving a quotation, you must include the page number, (Morgan, 1998, p. 123)

For two authors use 'and' (not &): (Morgan and Rowland, 1999)

For more than two authors, list all authors on first citing, subsequently, (Morgan *et al.*, 1999)

## References:

### 1. Book reference

Bernstein, B. (1996) *Pedagogy, Symbolic Control and Identity: Theory, Research, Critique*. London: Taylor and Francis.

### 2. Article reference

Chapman, A. (1995) 'Intertextuality in school mathematics: the case of functions.' *Linguistics and Education* 7(2), 243-262.

### 3. Chapter in edited book

Durkin, K. and Shire, B. (1991) 'Lexical ambiguity in mathematical contexts.' In K. Durkin and B. Shire (eds.), *Language in Mathematical Education: Research and Practice* (pp. 71-84). Buckingham: Open University Press.

Every work in the references should have been cited in the text i.e. don't give a 'bibliography' of books etc that you read but choose not to use in the essay.

## Referencing websites:

If you take material from a website, it needs to be referenced. It is referenced in the same way as a book, chapter, article etc. except you must note the URL (website address) *and when you accessed the website*. For example,

Murphy, G. (1999) 'Salazar and Modern Portugal'. <http://www.politicportugal/tbc/tcg-bt/info> (accessed 14.05.2000).

[note the date that you accessed it - gives some idea of currency]

Cite as Murphy (1999)

If there is no obvious author, then begin with the name of the website, not forgetting the access date. List at the end of your references.

Cite as www.xyzetc

Generally - official (eg DfES) websites are regarded as reliable sources of data, policies etc. But reports of research findings (even on university websites) are less reliable as evidence, because they don't have to be peer-reviewed - anyone can put stuff on the web. Of course, some published material is also posted on the web (eg some Ofsted reports) – in which case give the reference for the paper publication.

## Using quotations

1. Give short quotations in double inverted commas "...", immediately followed by the citation, with page number. Quotations that run to more than two lines should be indented and displayed, *without* inverted commas. For example:

Induction moves upward, and deduction downwards, on the same stair [...] Deduction descends steadily and methodically, step by step: Induction mounts by a leap which is out of the reach of method. She bounds to the top of the stairs at once [...] (Whewell, 1858, p. 114)

2. Are all your quotations copied accurately? Have you given the page number from which the quotation is taken?
3. Often a quotation includes extraneous matter making it less succinct than it otherwise need be. Cuts can be made by inserting "..." in place of the removed material (e.g. see the above quotation from Whewell). These cuts should not be made in order to distort meaning, but only to get to the heart of the matter.

4. Have you signalled the introduction of a quotation smoothly? Don't just 'parachute in' a quote, out of the blue. Generally, you will need to use introductory phrases such as "As Thompson points out, .." (alternatively, adds, argues, claims, remarks, replies etc). Only rarely can a quotation 'speak for itself': you need to provide a context for it.
5. It is sometimes better to *paraphrase* authors' ideas yourself rather than simply quote them, in order to exhibit a mark of independence. However, you must make sure that you present the gist of their ideas faithfully, and not paraphrase merely to suit your argument.

#### Miscellany

1. Avoid over-long sentences. Use commas with care; to break up sentences and clarify their meaning. Notice how other writers (good ones!) use commas.
2. When you cite or refer to an author, just give their surname(s), not their title (i.e. NOT Dr Jones, Dr Margaret Jones or even Professor Jones). However, you might sometimes include someone's first name when you *first* introduce them (although it's hard to give rules about when that is appropriate) e.g. "In a seminal article on methods of calculation, Stuart Plunkett argues that etc. Later, Plunkett (1986) proposed three categories ...".
3. Use of inverted commas: when do you use “..” rather than ‘...’ and *vice versa*? There is no universal practice, but we recommend the following. Use “...” only for quotations, however brief. For example, “The Cockcroft Report proposed the communicative power of mathematics as a “principal reason” for teaching it (HMSO, 1982, p. 1).” [The outer “...” above are there because we're quoting from an imaginary essay!]. Use ‘...’ either to *refer* to a word or phrase (e.g. “The term ‘ability’ is a difficult one to define”) or to suggest ‘so-called’, perhaps to hint that it is not a word that you would use yourself (e.g. “For more than a decade, teachers have been obliged to ‘deliver’ the subjects of the National Curriculum at the expense of topic work and cross-curricular themes.”).

#### Final advice

1. It's useful to use subheadings in your essay (and most journal articles do). These can be helpful signposts to the reader, and give a sharper focus to each section.
2. As a rule, write in the first person singular (“In this essay, I shall consider different ideas about etc”) rather than the third (“In this essay, the author will consider etc”) or the passive (“In this essay, different ideas about etc will be considered”). Third person writing is thought by some to be more detached, but runs a big risk of seeming pretentious. You will know that much written mathematical and scientific exposition is in the first person plural (“We begin by defining two transformations of the plane”). Much has been written about this phenomenon! But it's not appropriate in a mathematics education essay.
3. *Print out drafts* and read them yourself – critically. It's harder to detect errors on-screen.
4. Get a friend to read your essay when it's near to completion. They won't need to know anything about mathematics education to be able to pick out errors that you just can't see, because you've read it too many times.

And finally, just to please/humour your tutors: please don't use 'maths' for 'mathematics' (unless you want to offend Tim) or 'kids' for 'children' (unless you want to offend Peter).

## MATHEMATICS EDUCATION COURSE READINGS 2008-09

*NB: Forenames (Margaret etc) have been listed below by way of introduction to the authors. In lists of references, you should give initials only e.g. Brown, M. (1999)*

### **Block 1: Contexts**

Geoffrey Howson (1982) *A History of Mathematics Education in England* [Chapter 1: Robert Recorde]. Cambridge: Cambridge University Press.

Margaret Brown (1999) 'Swings of the Pendulum'. In I. Thompson (Ed.) *Issues in Teaching Numeracy in Primary Schools*. Buckingham: Open University Press, pp. 3-16

### **Block 2: Challenges to Practice**

Stuart Plunkett (1979) 'Decomposition and all that rot'. *Mathematics in School* 8(3) pp. 2-5

Bob Burn and NG Wood (1995) 'Teaching and learning mathematics in Higher Education'. *Teaching Mathematics and its Applications* 14(1) pp. 28-33.

### **Block 3: Why learn mathematics?**

Paul Andrews (1998) 'Peddling the Myth'. *Mathematics in School* 27(2), pp. 2-4

Peter Huckstep (1999) 'How can mathematics be useful?'. *Mathematics in School* 28(1) pp. 15-17

Richard Smith (2000) 'Insight and assurance'. In S. Bramall and J. White (eds), *Why Learn Maths?* (pp. 15-25). London: Institute of Education, University of London.

### **Block 4: Thinking and learning 1**

Richard Skemp (1976) 'Relational understanding and instrumental understanding'. *Mathematics Teaching* 77, pp. 20-26

Tony Harries and Mike Spooner (2000) 'Perspectives on the teaching and learning of mathematics'. Pp. 12-30 of *Mental Mathematics for the Numeracy Hour*. London: David Fulton.

### **Block 5: Arithmetic**

Ian Thompson (1999) 'Getting your head around mental calculation'. In I. Thompson (Ed) *Issues in Teaching Numeracy in Primary Schools* (pp.145-156) Buckingham: Open University Press.

Cathy Smith (1999) 'Pencil and paper numeracy'. *Mathematics in School* 28(5) pp. 10-13.

### **Block 6: Thinking and learning 2**

Martin Hughes (1986) *Children and Number* [Chapter 4]. Oxford: Blackwell.

Tim Rowland (1999) 'The clinical interview: conduct and interpretation'. In Zaslavsky, O. (Ed.) *Proceedings of the 23rd Conference of the International Group for the Psychology of Mathematics Education*. Vol. 4, pp. 129-136. Haifa, Israel: Israel Institute of Technology.

### **Block 7: Comparative Mathematics Education**

Albert Beaton and David Robitaille (1999) 'An overview of the Third International Mathematics and Science Study'. In Kaiser, G., Luna, E., Huntley, I. (Eds) *International Comparisons in Mathematics Education* (pp. 30-47) London: Falmer.

Carol Murphy (2003). 'A theoretical comparison of the teaching of mental calculation strategies in England and the Netherlands'. In S.Pope and O.McNamara (eds) *Research in Mathematics Education*, Vol 5 (pp. 123-137). London: British Society for Research into Learning Mathematics.

### **Block 8: Proof**

Melissa Rodd and John Monaghan (2001) 'School mathematics and mathematical proof'. In L. Haggarty (Ed) *Aspects of Teaching Secondary Mathematics* (pp. 71-90) Milton Keynes, Open University Press.

Elena Nardi and Paula Iannone (2006) *How To Prove It: a brief guide for teaching Proof to mathematics undergraduates*. (pp. 5-22) Commissioned by the Higher Education Academy. <http://mathstore.ac.uk/publications/index.shtml>



## SOME EXAMPLES OF ISSUES IN MATHEMATICS EDUCATION

Note that the issues listed below are a selection of possibilities, in alphabetical order, and that they are not all addressed to the same extent in the readings.

Aims and purposes of mathematics education

Assessment

Comparative education - practices in different countries

Inclusion, equity and social justice

Explanation and proof

Investigative approaches

Language and mathematics

Mathematics in the home

Methods of (approaches to) calculation

'Numeracy' and mathematics

Particular topics e.g. arithmetic, geometry, probability

Problem solving

Situated mathematics - in the workplace etc

Technology - computers, calculators

Textbooks

The mathematics curriculum

The nature of mathematics

Theories of knowledge

Theories of learning

Theories of teaching

Understanding in mathematics

SUBMITTED ESSAY FOR MATHEMATICS EDUCATION: PROPOSAL FORM

Complete the first part of this form before your first individual supervision. You *must* take a copy of it with you to that meeting with your supervisor to discuss your course assignment essay.

Name:

Proposed essay title:

To which theme(s) in the taught course (e.g. 'Thinking and Learning) will the essay most closely relate?

List the readings from the course which you would expect to refer to in the essay.

List any other relevant literature which you have already identified.

Approval sections (to be completed at the supervision before the end of Lent Term)

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[To be retained by the student]

Final approved title.

Signed (Supervisor)

Date:

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[To be detached and retained by the supervisor for Faculty records]

Student's name:

Approved title.

Supervisor:

Date