THE KNOWLEDGE QUARTET: A TOOL FOR DEVELOPING MATHEMATICS TEACHING

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ABSTRACT

This paper draws on videotapes of mathematics lessons prepared and conducted by pre-service elementary teachers towards the end of their initial training. A grounded theory approach to data analysis led to the identification of a 'knowledge quartet', with four broad dimensions, through which the mathematics-related knowledge of these beginning teachers could be observed in practice. We term the four units: foundation, transformation, connection and contingency. This paper describes how each of these units is characterised, and analyses a fragment of one of the videotaped lessons, showing how each dimension of the quartet can be identified in the lesson. We claim that the quartet can be used as a framework for lesson observation and for mathematics teaching development.

INTRODUCTION

The seven categories of teacher knowledge identified in the seminal work of Lee Shulman include three with an explicit focus on 'content' knowledge: subject matter knowledge, pedagogical content knowledge and curricular knowledge. Subject matter knowledge (SMK) is knowledge of the content of the discipline *per se* (Shulman, 1986, p. 9), consisting both of substantive knowledge (the key facts, concepts, principles and explanatory frameworks in a discipline) and syntactic knowledge (the nature of enquiry in the field, and how new knowledge is introduced and accepted in that community).

Pedagogical content knowledge (PCK) is particularly difficult to define and characterise, conceptualising both the link and the distinction between knowing something for oneself and being able to enable others to know it. PCK consists of "the ways of representing the subject which makes it comprehensible to others…[it] also includes an understanding of what makes the learning of specific topics easy or difficult …" (Shulman, 1986, p. 9). Curricular knowledge encompasses the scope and sequence of teaching programmes and the materials used in them.

An uninformed perspective on SMK in relation to mathematics teaching might be characterised by the statement that secondary teachers already have it and elementary teachers need very little of it. There is evidence from the UK and beyond to refute both parts of that statement (e.g. Ball, 1990a, b; Alexander, Rose and Woodhead, 1992; Ofsted, 1994; Ma, 1999). Ma, in particular, presents compelling evidence that the adequacy of elementary teachers' substantive and syntactic knowledge of mathematics, for their own professional purposes, cannot by any means be taken for granted.

This paper is located in a collaborative project involving researchers at three UK universities, under the acronym *SKIMA* (subject knowledge in mathematics). The conceptualisation of subject knowledge which informs the project and its relation to teaching has been detailed elsewhere (Goulding, Rowland and Barber, 2002). The research reported in this paper was undertaken in collaboration with two *SKIMA* colleagues, Peter Huckstep and Anne Thwaites. I shall sometimes use the pronoun 'we' in this text, in order to acknowledge their contribution.

The focus of this particular research is on the ways that teacher trainees' mathematics content knowledge - both SMK and PCK - can be observed to 'play out' in practical teaching during school-based placements. The research had multiple objectives, the first of which was to develop an empirically-based conceptual framework for productive discussion of mathematics content knowledge, between teacher educators, trainees and teacher-mentors, in the context of school-based placements. Such a framework would need to be manageable, and not overburdened with structural complexity. It would need to capture a number of important ideas and factors about content knowledge within a small number of conceptual categories, with an equally small set of easily-remembered labels for those categories. This path that we followed to achieve this objective is the subject of this paper.

We wish to clarify at the outset that whilst we see certain kinds of knowledge to be *desirable* for elementary mathematics teaching, we are convinced of the futility of asserting what a beginning teacher, or a more experienced one for that matter, *ought* to know. Our interest is in what a teacher *does* know and believe, and how opportunities to enhance knowledge can be identified. We believe that the framework that arose from this research – we call it the 'knowledge quartet' – provides a means of reflecting on teaching and teacher knowledge, with a view to developing both. We begin with a brief résumé of our purposes and methods.

PURPOSE OF THE RESEARCH

In the UK, most trainee teachers follow a one-year, postgraduate course leading to a Postgraduate Certificate in Education (PGCE) in a university education department. All primary (elementary) trainees are trained to be generalist teachers of the whole primary curriculum. Over half of the PGCE year is spent working in schools under the guidance of a school-based mentor. Placement lesson observation is normally followed by a review meeting between a school-based teacher-mentor and the student-teacher. On occasion, a university-based tutor will participate in the observation and the review. Research shows that such meetings typically focus heavily on organisational features of the lesson, with very little attention to mathematical aspects of mathematics lessons (Brown, McNamara, Jones and Hanley, 1999; Strong and Baron, 2004). The purpose of the research reported in this paper was to develop an empirically-based conceptual framework for the discussion of the role of trainees' mathematics SMK and PCK, in the context of lessons taught on the school-based placements. Such a framework would need to capture a number of important ideas and factors about content knowledge within a small number of conceptual categories, with a set of easily-remembered labels for those categories.

METHOD

This study took place in the context of a one-year PGCE course, in which 149 trainees followed a route focusing either on the 'lower primary' years (LP, ages 3-8) or the 'upper primary' (UP, ages 7-11). Six trainees from each of these groups were chosen for observation during their final school placement. Two mathematics lessons taught by each of these trainees were observed and videotaped, i.e. 24 lessons in total. We took a grounded theory approach to the data for the purpose of generating theory (Glaser and Strauss, 1967). In particular, we identified aspects of trainees' actions in the classroom that seemed to be significant in the limited sense that it could be construed to be informed by their mathematics SMK or PCK. These were grounded in particular moments or episodes in the tapes. This inductive process generated a set of 18 codes. This was valuable from the research perspective, but presented us with a practical problem. We intended to offer our findings to colleagues for their use, as a framework for reviewing trainees' mathematics content knowledge from evidence gained from classroom observations of teaching. We anticipate, however, that 18 codes is too many to be useful for a one-off observation. Our resolution of this dilemma was to group them into four broad, super-ordinate categories, or 'units', which we term 'the knowledge quartet'.

FINDINGS

We have named the four units of the knowledge quartet as follows: foundation; transformation; connection; contingency. Each unit is composed of a small number of cognate subcategories. For example, the third of these, *connection*, is a synthesis of four of the original 18 codes, namely: *making connections; decisions about sequencing; anticipation of complexity,* and *recognition of conceptual appropriateness.* Our scrutiny of the data suggests that the quartet is comprehensive as a tool for thinking about the ways that subject knowledge comes into play in the classroom. However, it will become apparent that many moments or episodes within a lesson can be understood in terms of two or more of the four units; for example, a **contingent** response to a pupil's suggestion might helpfully **connect** with ideas considered earlier. Furthermore, it could be argued that the application of subject knowledge in the classroom *always* rests on **foundational** knowledge. Drawing on the extensive range of data from the 24 lessons, we offer here a brief conceptualisation of each unit of the knowledge quartet.

Foundation

The first member of the quartet is rooted in the foundation of the trainees' theoretical background and beliefs. It concerns trainees' knowledge, understanding and ready recourse to their learning *in the academy*, in preparation (intentionally or otherwise) for their role in the classroom. It differs from the other three units in the sense that it is about knowledge possessed, irrespective of whether it is being put to purposeful use. This distinction relates directly to Aristotle's account of 'potential' and 'actual' knowledge. "A man is a scientist … even when he is not engaged in theorising, provided that he is capable of theorising. In the case when he is, we say that he is a scientist in actuality." (Lawson-Tancred, 1998, p. 267). Both empirical and theoretical considerations have led us to the view that the other three units flow from a foundational underpinning.

A key feature of this category is its *propositional* form (Shulman, 1986). It is what teachers learn in their 'personal' education and in their 'training' (pre-service in this instance). We take the view that the possession of such knowledge has the potential to inform pedagogical choices and strategies in a fundamental way. By 'fundamental' we have in mind a rational, reasoned approach to decision-making that rests on something other than imitation or habit. The key components of this theoretical background are: knowledge and understanding of mathematics *per se*; knowledge of significant tracts of the literature and thinking which has resulted from systematic enquiry into the teaching and learning of mathematics; and espoused beliefs about mathematics, including beliefs about why and how it is learnt.

In summary, this category that we call 'foundation' coincides to a significant degree with what Shulman (1987) calls 'comprehension', being the first stage of his six-point cycle of pedagogical reasoning.

Transformation

The remaining three categories, unlike the first, refer to ways and contexts in which knowledge is brought to bear on the preparation and conduct of teaching. They focus on knowledge-in-action as *demonstrated* both in planning to teach and in the act of teaching itself. At the heart of the second member of the quartet, and acknowledged in the particular way that we name it, is Shulman's observation that the knowledge base for teaching is distinguished by " … the capacity of a teacher to *transform* the content knowledge he or she possesses into forms that are pedagogically powerful" (1987, p. 15, emphasis added). This characterisation has been echoed in the writing of Ball (1988), for example, who distinguishes between knowing some mathematics 'for yourself' and knowing in order to be able to help someone else learn it. As Shulman indicates, the presentation of ideas to learners entails their re-presentation (our hyphen) in the form of analogies, illustrations, examples, explanations and demonstrations (Shulman, 1986, p. 9). Our second category, unlike the first, picks out behaviour that is directed towards a pupil (or a group of pupils), and which follows from deliberation and judgement informed by foundation knowledge. This category, as well as the first, is informed by particular kinds of literature, such as the teachers' handbooks of textbook series or in the articles and 'resources' pages of professional journals. Increasingly, in the UK, teachers look to the internet for bright ideas and even for ready-made lesson plans. The trainees' choice and *use of examples* has emerged as a rich vein for reflection and critique. This includes the use of examples to assist concept formation, to demonstrate procedures, and the selection of exercise examples for student activity.

Connection

The next category binds together certain choices and decisions that are made for the more or less discrete parts of mathematical content – the learning, perhaps, of a concept or procedure. It concerns the *coherence* of the planning or teaching displayed across an episode, lesson or series of lessons. Mathematics is notable for its coherence as a body of knowledge and as a field of enquiry, and the cement that holds it together is reason. The pursuit of coherence and mathematical connections in mathematics pedagogy has been stimulated recently by the work of Askew, Brown, Rhodes, Wiliam and Johnson (1997); of six case study teachers found to be highly effective, all but one gave evidence of a 'connectionist' orientation. The association between teaching effectiveness and a set of articulated beliefs of this kind lends a different perspective to the work of Ball (1990b) who also strenuously argued for the importance of connected knowledge for teaching.

In addition to the integrity of mathematical content in the mind of the teacher and his/her management of mathematical discourse in the classroom, our conception of coherence includes the *sequencing* of topics of instruction within and between lessons, including the ordering of tasks and exercises. To a significant extent, these reflect deliberations and choices entailing not only knowledge of structural connections within mathematics itself, but also awareness of the relative cognitive demands of different topics and tasks.

Contingency

Our final category concerns the teacher's response to classroom events that were not anticipated in the planning. In some cases it is difficult to see how they could have been planned for, although that is a matter for debate. In commonplace language this dimension of the quartet is about the ability to 'think on one's feet': it is about *contingent action.* The two constituent components of this category that arise from the data are the readiness to *respond to children's ideas* and a consequent preparedness, when appropriate, to *deviate from an agenda* set out when the lesson was prepared. Shulman (1987) proposes that most teaching begins from some form of 'text' - a textbook, a syllabus, ultimately a sequence of planned, intended actions to be carried out by the teacher and/or the students within a lesson or unit of some kind. Whilst the stimulus - the teacher's intended actions - can be planned, the students' responses can not.

Brown and Wragg (1993) group listening and responding together in a taxonomy of 'tactics' of effective questioning. They suggest that 'responding' moves are the lynch pins of a lesson, important in the sequencing and structuring of a lesson, and observe that such interventions are some of the most difficult tactics for newly qualified teachers to master. The quality of such responses is undoubtedly determined, at least in part, by the knowledge resource available to the teacher. For example, Bishop (2001, pp. 95-96) recounts a nice anecdote about a class of 9- and 10-year-olds who were asked to give a fraction between ½ and ¾. One girl answered $^{2}/_{3}$, "because 2 is between the 1 and the 3, and on the bottom the 3 lies between the 2 and the 4". Bishop asks his readers how they might respond to the pupil. It is relevant here to suggest that such a response might be conditioned by whether they were aware of Farey sequences and mediants, or what heuristics were available to them to explore the generalisation inherent in the pupil's justification.

CHLOË'S LESSON

We now proceed to show how this theoretical construct, the knowledge quartet, might be applied, by detailed reference to a 14-minute portion of one of the 24 videotaped lessons. The trainee in question, Chloë, was teaching a Year 1/2 (pupil age 5-7) class a particular strategy for mental subtraction. By focusing on this vignette we aim to maximise the possibility of the reader's achieving some familiarity with the scenario, with Chloë and a few of the children in her class. What is lost, of course, is any sense of how the quartet might inform reflection on the rest of her lesson. On the other hand, the passage we have selected would be, in itself, a valuable focus for some useful reflection in the post-lesson mentoring discussion.

Conforming to the English *National Numeracy Strategy* (NNS) guidance (DfEE, 1999), Chloë segments the lesson into three distinctive and readily-identifiable phases: the *mental and oral starter*; the *main activity* (an *introduction* by the teacher, followed by *group work,* with tasks differentiated by pupil ability); and the concluding *plenary*. The learning objective stated in Chloë's lesson plan is: "Children should be able to subtract 9, 11, 19 and 21 using the appropriate strategies". The lesson begins with a three-minute *mental and oral starter,* in which Chloë asks a number of questions such as 'How many must I add to 17 to make 20?', 'How many more than 7 is 10?', designed to test recall of complements of 10 and 20. There follows a 14-minute *introduction* to the *main activity*. Chloë reminds the class that in their previous lesson (which was taught by her mentor) they added 9, 11, 19 and 21 to various 1-digit and 2-digit whole numbers. Chloë demonstrates how to subtract these same numbers by subtracting 10 or 20 first, then adding or subtracting 1. She has a large, vertically-mounted 1-100 square, and models the procedure, moving a counter vertically and horizontally on the hundred square. She calls on children to assist her as 'teachers' in the demonstration. At the end of the demonstration, Chloë lists an example of each of the four subtractions on a whiteboard. The class then proceeds to 23 minutes' seatwork on differentiated worksheet exercises that Chloë has prepared. The 'more able' children subtract 19 and 21, the others subtract 9 and 11. Finally, she calls them together for a four-minute *plenary*, in which they consider 30 – 19 and 43 – 21 together.

Chloë's Lesson and the Knowledge Quartet

We now home in on the introduction to the main activity, to see how it might be perceived through the lens of 'the knowledge quartet'. This is typical of the way that the quartet can be used to identify for discussion various matters that arise from the lesson observation, and to structure reflection on the lesson. Some possibilities for discussion with the trainee, and for subsequent reflection, are flagged below thus: **Discussion point**. We emphasise that the process of *selection* in the commentary which follows has been extreme. Nevertheless, it offers a realistic agenda for a typical, time-constrained post-lesson review meeting.

Foundation: Chloë's lesson plan refers to "appropriate strategies" for subtracting four near-multiples of 10, without recording what strategies she has in mind. It becomes clear that she will emphasise mental, sequential strategies, perhaps with some use of informal jottings (DfEE, 1999, p. 2/4). This is very much in keeping with the *National Numeracy Strategy,* which, following the Dutch RME (Realistic Mathematics Education) approach, emphasises mental calculation methods in the early grades. Sequential (or cumulative) strategies for two-digit addition and subtraction begin with one number (for subtraction, the minuend) and typically move up or down the sequence of integers in tens or ones. Split-tens methods, by contrast, partition both numbers into tens and units and operate on the two parts separately, before re-combining (e.g. Anghileri, 2000, pp. 62-65). The objective of the previous lesson (on adding near-tens) and the current one is taken directly from the NNS *Framework* (DfEE, 1999) teaching programme for Year 2:

> Add/subtract 9 or 11: add/subtract 10 and adjust by 1. Begin to add/subtract 19 and 21: add/subtract 20 and adjust by 1. (p. 3/10)

These objectives are clarified by examples later in the *Framework*; such as

> 58+21=79 because it is the same as 58+20+1; 70-11=59 because it is the same as 70-10-1
>
> 24-9=15 because it is the same as 24-10+1; 35+19=54 because it is the same as 35+20-1 (p. 4/35)

The superficial similarity in these examples; captured in the NNS objective immediately above, is, we would suggest, deceptive. The differences between them can be articulated in terms of what Marton and Booth (1997) call 'dimensions of variation'. The dimensions in this case bring with them different kinds and levels of complexity, as follows.

Dimension 1: Addition or subtraction. In general terms, it might be thought that subtraction is the more demanding. Indeed, the first lesson of the two had dealt exclusively with addition, the second with subtraction.

Dimension 2: Near multiples of 10 or 20. Again, it seems reasonable to anticipate that adding/subtracting 20 is the more demanding. Indeed, Chloë has explicitly planned for the lower-attaining groups of pupils to work exclusively with 9 and 11.

Dimension 3: One more or one less than 10/20. Addition and subtraction of 11/21 entail a sequence of actions in the *same* direction i.e. aggregation or reduction; whereas 9/19 require a *change* of direction for the final unit i.e. compensation. Research confirms what might be expected, that the latter is less spontaneous and more demanding (e.g. Heirdsfield, 2001). Indeed, the compensation strategy for adding/subtracting 9 is, in lay terms, a 'trick'.

Discussion point: what considerations determined Chloë's choice of worksheet problems for the two 'ability' groups in the class?

Transformation. We pick out two factors for consideration relating to this dimension of the quartet (as usual, bearing in mind that they are underpinned by foundational knowledge). First, Chloë's use of the 100 square as a model or representation of the sequence of two-digit positive integers. The 100 square is useful for representing ordinal aspects of the sequence, though with some discontinuities at the 'ends' of the rows, and particularly for representing the place-value aspects, although a 0-99 square arguably does this better (Pasternack, 2003). Chloë makes full use of the 100 square in her exposition, but is frequently dismissive of children's use of the spatial language that it invites. For example, subtracting 9 from 70, she places the counter on 70:

Chloë:	Right, there's 70. […] From 70 I want to take away nine. What will I do? Rebecca?
Rachel:	Go up one.
Chloë:	No, don't tell me what I'm gonna go up or move, tell me what I actually do.
Rachel:	Take away one.
Chloë:	Take away one to take away nine? No. Remember when we added nine we added ten first of all, so what do you think we might take away here? Sam.
Simon:	Ten.

This would seem to relate to the format of the NNS examples (above), which she follows in four 'model' solutions that she writes for reference on the board, e.g.

70 - 9 = ?, 70 - 10 + 1 = 61

Somewhat surprisingly, the children are forbidden to use 100 squares when they do the worksheet exercises. Chloë refuses a request from one child for a "number square”, saying, “I want you to work them out all by yourselves”. In fact, there is nothing in Chloë’s lesson plan to indicate that she had intended to use the 100 square in her demonstration.

> **Discussion point**: What led Chloë to use the 100 square? What are its potential affordances - and constraints - for calculation relative to the symbolic recording in the NNS examples? Had she considered using an empty number line (e.g. Rousham, 2003) as an alternative way of representing the numbers and their difference, of clarifying when compensation is necessary, and why?

The other aspect of transformation that we select here concerns Chloë’s choice of examples. As we have observed, this has emerged as a rich vein for reflection and critique in every one of the 24 videotaped lessons. Space considerations restrict us to mentioning just one, in fact the first chosen to demonstrate subtraction, following the initial review of addition. Chloë chooses to subtract 19 from 70. We have already argued that subtracting 11 and 21 would be a more straightforward starting point. Moreover, 70 is on the extreme right boundary of the 1 to 100 square. After moving up two squares to 50, there is no ‘right one’ square: it is then necessary to move down and to the extreme left of the next row, so the neat ‘knights move’ is obscured, and the procedure unnecessarily complicated. We note that one of the NNS *Framework* examples (above) is 70 - 11, and that all four of Chloë’s whiteboard template examples were of the form 70 - *n*.

> **Discussion point**: Was Chloë aware in-the-moment of the complication mentioned above, or did she anticipate it in her planning? Did the symbolic form in her written plan (70 - 20 +1) perhaps obscure the consequences of her using the 100 square for this calculation?

Connection. Chloë makes explicit links with the previous lesson on adding near-multiples of 10, and reviews the relevant strategies at the start of this one. Her oral and mental starter, on complements to 10 and 20, essentially focuses on the concept of subtraction as comparison, whereas the strategy taught in the main activity is on change-separate, or ‘take away’, subtraction (Carpenter and Moser, 1983). Procedures associated with the two concepts tend to be based on strategies for counting on and counting back respectively (*ibid*.). Arguably Chloë could have encouraged some flexibility in the choice of such procedures , whereas she chose to prescribe exclusively forms of counting back in the main activity. The effect of her approach to differentiation for the different groups was to emphasise the similarity between 9 and 11 (needing an initial subtract-10) and between 19 and 21 (subtract-20), when the pairing of 11 and 21 (consistent reduction) and 9 and 19 (needing compensation) was an alternative form of connection.

Given her use of the 100 square to demonstrate the strategies, there was scope for some discussion of the links between vertical and horizontal spatial movements on the board and the tens-ones structure of the numbers under consideration. As we have remarked, she actively discouraged children’s reference to the spatial analogue. It seemed that her attention was on conformity at the expense of flexibility and meaning-making.

> **Discussion point**: discussion could usefully focus on the two subtraction concepts, how they relate to the first two phases of the lesson, and whether comparison strategies might offer useful alternatives to 'take-way with compensation', in the case of subtracting 9/19.

Contingency. A key component of our conceptualisation of this dimension of the quartet relates to how the teacher responds to unexpected or deviant ideas and suggestions from children in the lesson. There are no compelling distractions from Chloë's planned agenda for the lesson in this episode, although the child's question about using the number squares for the exercises might be a case in point. Various children's use of up/down language on the 100 square, to which we have already referred, might have been usefully explored rather than dismissed. A similar opportunity presented itself when, in the review of adding 9 at the beginning of the lesson, Chloë invites one of the pupils to demonstrate:

Chloë:	Show the class how you add ten and take away one on a number square. What's the easy way to add ten on a number square? Cameron.
Cameron:	Go diagonally.
Chloë:	Not diagonally. To add ten you just go…
Cameron:	Down.

No further reference is made to Cameron's diagonal proposal, although his elegant use of vocabulary alone is surely worth a moment's pause. It is true that his initial suggestion is not, strictly, a correct answer to her "add ten" question. It does, however, offer a nice spatial way of thinking about adding 9 - and adding 11 too - and suggests that Chloë's mentor may have stressed it in the previous lesson. Indeed, the fact that adding 9 corresponds to a diagonal south-west move might usefully connect to the insight that subtracting 9 would necessitate a north-east move, and the consequent need to *add* one after subtracting 10. It would seem that Chloë is too set on her own course to explore the possibilities offered by remarks such as Cameron's.

> **Discussion** point: Did Chloë recall Cameron's suggestion? If so, how did she feel about it at the time, and how might she have responded differently?

It is important to add that the second question in this proposed discussion point is not intended as a thinly-veiled rebuke or correction: there are often very good reasons for teachers sticking to their chosen path. The purpose of the question is to raise awareness of the fact that an opportunity was presented, and that a different choice could have been made. We also reiterate that a single event or episode can frequently be considered from the perspective of two or more dimensions of the quartet, as demonstrated in our commentary.

FINAL COMMENTS AND CAVEATS

In this paper, we have introduced 'the knowledge quartet' and shown its relevance and usefulness in our analysis of part of Chloë's lesson with a Year 1/2 class. We have a manageable framework within which to discuss actual, observed teaching sessions with trainees and their mentors. These groups of participants in initial teacher preparation, as well as our university-based colleagues, need to be acquainted with (and convinced of the value of) the quartet, and to be familiar with some details of its conceptualisation, as described in this paper. Within the last year we have taken steps towards this familiarisation in the context of our own university's pre-service elementary and middle school teacher education programmes. The four dimensions of the knowledge quartet have been used as a framework for lesson observation and reflection. Initial indications are that this development has been well received by mentors, who appreciate the specific focus on *mathematics* content and pedagogy. They observe that it compares favourably with guidance on mathematics lesson observation from the NNS itself, which focuses on more generic issues such as "a crisp start, a well-planned middle and a rounded end. Time is used well. The teacher keeps up a suitable pace and spends very little time on class organisation, administration and control." (DfEE, 2000, p.11).

It is all too easy for an observer to criticise a novice teacher for what they omitted or committed in the high-stakes environment of a school placement, and we would emphasise that the quartet is intended as a tool to *support* teacher development, with a sharp and structured focus on the impact of their SMK and PCK on teaching. Indications of how this might work are explicit in our analysis of Chloë's lesson. We have emphasised that our analysis has been *selective*: we raised for attention some issues, but there were others which, not least out of space considerations, we chose not to mention. The same would be likely to be true of the review meeting - in that case due to *time* constraints, but also to avoid overloading the trainee with action points. Such a meeting might well focus on a lesson fragment, and on only one or two dimensions of the knowledge quartet for similar reasons.

Any tendency to descend into deficit discourse is also tempered by consideration of the wider context of the student teacher's experience in school. In the novice teacher we see the very beginnings of a process of reconciliation of pre-existing beliefs, new 'theoretical' knowledge, 'practical' advice received from various quarters, in the context of highly-pressured, high-stakes school-based placements. There is also good evidence (e.g. Hollingsworth, 1988; Brown, Mcnamara, Jones, and Hanley, 1999) that trainees' concern for pupil learning is often eclipsed by their anxieties about timing, class management and pupil behaviour.

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