

## **Nuffield Seminar Series: Mathematical Knowledge in Teaching**

### **Paper for Seminar 6: Issues for future research: Tim Rowland and Fay Turner**

In the light of the recent interim report of the Williams review (May 2008) into mathematics teaching in primary schools and early years settings, we see an urgent need for two related strands of research to be reviewed and developed in a way that can be communicated to policy makers and those involved in ITE and PPD provision:

- research to strengthen understanding of what constitutes ‘deep’ content knowledge for teaching primary mathematics;
- research into how such knowledge may be developed through ITE and PPD.

#### **Research to strengthen understanding of what constitutes deep content knowledge for teaching primary mathematics**

The account of knowledge for mathematics teaching in the interim report is rather blunt and simplistic. Existing research suggests that deep knowledge for teaching primary mathematics involves:

- both SMK and PCK (Shulman, 1986);
- both substantive and syntactic knowledge (Schwab, 1978);
- relational rather than instrumental understanding of mathematical ideas (Skemp, 1978);
- connectedness (Askew, Brown, Rhodes, Johnson and Wiliam, 1997; Chi, Glaser and Rees, 1982; Leinhardt and Smith, 1985);
- profound understanding of fundamental mathematics (Ma, 1999);
- both common content knowledge and specialised content knowledge (Ball, Hill and Bass, 2005);
- appropriate knowledge observable in teaching as conceptualised by the four dimensions of the Knowledge Quartet; foundation, transformation, connections and contingency (Rowland, Huckstep and Thwaites, 2005).

There is a great deal of overlap and complimentarity between these ideas, but seemingly little contradiction. A first step might be to attempt an integration of these ideas and any others that are thought to be helpful in developing a clearer account for practitioners and policy makers of ‘deep subject knowledge for teaching mathematics’. This would then support the next stage of research into specific knowledge ‘needed’ to teach primary mathematics. There is existing research in this field e.g. argumentation and proof (Yackel, 2002; Stylianides and Ball, in press; Knuth, 2002) and fractions (An, Kulm, and Wu, 2004; Borko, Eisenhart, Brown, Underhill, Jones and Agard, 1992). Such research should be reviewed in order to identify gaps in areas of primary mathematics and related to the comprehensive definition of deep subject knowledge.

#### **Research into how deep knowledge of mathematics may be developed through ITE and PPD**

Research indicates that changes in knowledge during ITE courses are ‘small and insignificant’ (Carré and Ernest, 1993). When trainee teachers take up teaching posts, it is the situation in which they work

that has the greatest influence in changing their mathematical knowledge for teaching (Brown, McNamara, Jones and Hanley 1999; McNamara, Jaworski, Rowland, Hogden and Prestage, 2002).

Socio-cultural theories of learning seem to offer a useful way forward in identifying how mathematical knowledge for teaching might be developed. If we cannot make sustainable progress in developing deep mathematical knowledge during ITE courses, then we must consider how this might be achieved through PPD. If teacher learning is *situated* (Cobb, Yackel and Wood, 1991) and takes place within *communities of practice* (Lave and Wenger, 1991) then research should be carried out into the nature of these CoPs, and how they promote or inhibit development of deep knowledge of mathematics. Brown, McNamara, Jones and Hanley (1999) suggested that development facilitated by integration into CoPs may not lead to mathematics teaching that is consistent with contemporary views of effective teaching (Cockcroft, 1982; NCTM, 2000).

It is therefore appropriate to ask how mathematics educators might *intervene* in the learning ecology of such CoPs. Jaworski (2006) offers a way of thinking about such intervention by modifying Wenger's (1998) notion of *alignment*, in relation to development of professional identity, to that of *critical alignment* allowing for critical evaluation of the practices of the community while aligning oneself with it. Such *critical alignment* may be seen as a means to develop deeper content knowledge in teachers and might be promoted in a number of different ways.

Researchers have suggested that content knowledge might be developed by:

- using the Knowledge Quartet framework as a 'tool' for helping teachers reflect on their mathematical content knowledge as revealed through their teaching (Turner 2008);
- identification of areas of content knowledge needing development using a multiple choice questionnaire of mathematics knowledge (Ball et al, 2005);
- lesson study (Lewis, Perry and Murata, 2006);
- development of Profound Understanding of Fundamental Mathematics (PUFM) through a 'culture of discussion' (Ma, 1999);
- personal participation in mathematics (Watson, 2008)

Any of these may be carried out within *communities of practice* - which might then be encouraged to become *communities of inquiry* (Jaworski, 2006). A research agenda might therefore be to focus interventions by mathematics educators on work within both existing communities of practice and those created for a particular programme or project. Research into the effectiveness of such interventions should continue to inform how communities of practice can best be utilised to develop deep knowledge in all teachers.

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