MICHIGAN RESEARCH ON DEVELOPING A PRACTICE-BASED THEORY OF CONTENT KNOWLEDGE OF TEACHING

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1. INTRODUCTION

This paper summarises the efforts of Michigan University's mathematics education research team (School of Education) to develop a theory of content knowledge of teaching, building on Shulman's notion of Pedagogical Content Knowledge (PCK). Over the last fifteen years the team has developed two projects, the Mathematics Teaching and Learning to Teach Project (MTLT) and the Learning Mathematics for Teaching Project (LMT). The aim was to develop an empirical approach to identify what teachers need to know in order to teach mathematics effectively. Particularly, the team has been devising instruments to measure both teachers' Subject Matter Knowledge (SMK) and PCK, and that can give good predictions of classroom effectiveness and students learning. Moreover, these measures helped the team to develop a practice-based theory of content knowledge for teaching by testing and refining categories of knowledge needed to teach mathematics effectively.

2. THE MATHEMATICS TEACHING AND LEARNING TO TEACH PROJECT (MTLT) AND THE LEARNING MATHEMATICS FOR TEACHING PROJECT (LMT)

2.1 The rationale behind the development of the MTLT and LMT projects

In the mid 1980's Shulman and his colleagues initiated a new wave of interest in research on teachers' knowledge and teaching. They suggested that research of the day devoted little attention to the role that teacher content knowledge might have on their teaching. Thus, in his 1986 paper Shulman proposed a new domain of teacher knowledge, which he termed Pedagogical Content Knowledge.

In the 1986 paper PCK is referred to a subcategory of teachers' content knowledge (the other two being SMK and curriculum knowledge). PCK 'goes beyond knowledge of subject matter per se to the dimension of subject matter knowledge for teaching…' (Shulman, 1986, p.9), meaning that PCK includes the interpretations, examples and applications that teachers use in order to make subject matter comprehensible to students (Shulman, 1986). In Shulman's (1987) later work PCK was conceptualised as one of the seven categories that described teachers' knowledge (the other six being content knowledge, general pedagogical knowledge, curriculum knowledge, knowledge of learners and their characteristics, knowledge of educational contexts, and knowledge of educational purposes and values). In both papers, as a concept, PCK, suggested that is not only knowledge of the content that is central to teaching but also a special type of professional knowledge for teaching is needed by teachers in order to teach effectively.

Shulman's conceptualisation of PCK quickly became the standard definition, and many researchers used it in their studies. However, Ball, Thames and Phelps (2007) argue that,

while the term pedagogical content knowledge is widely used, its potential remains insufficiently exploited. Its essential conceptual core has too often taken for granted, and hence remains poorly differentiated and specified (p.4)

In other words Ball et al (2007) argue that in the past years the term PCK was used in many different ways in the field of teacher knowledge in terms of what it includes and in how is used to relate content knowledge to the practice of teaching. Moreover, in many cases researchers used logical arguments to support their claims about the existence of different categories of teachers' content knowledge and only few studies tried to develop measures to test definitions of the nature of the different categories of content knowledge.

Identifying this gap in the research the Michigan's mathematics education research team designed an empirical approach to identify and understand the knowledge needed for teaching mathematics effectively. Their aim was to develop a practice-based theory of content knowledge for teaching by developing instruments to measure both teachers' SMK and PCK. These measures helped the team to test and refine categories of knowledge needed to teach mathematics effectively, building on Shulman's notion of PCK. The work of the team started 15 years ago and is still ongoing. They designed two research projects, the MTLT and the LMT. The main focus of both projects was to identify what teachers need to know and be able to do in order to teach mathematics effectively.

2.2 The Mathematics Teaching and Learning to Teach Project.

The MTLT project investigates the mathematical knowledge and skills that are involved in mathematics teaching. The team decided to focus on what teachers do while teaching (Ball, Hill and Bass, 2005). By teaching they mean:

everything that teachers do to support the instruction of their students…the interactive work of teaching lessons in classrooms, and all the tasks that arise in the course of that… Each of these tasks involves knowledge of mathematical ideas, skills of mathematical reasoning…fluency with examples, and thoughtfulness about the nature of mathematical proficiency (Ball et al 2005, p.17)

The team used qualitative methods to collect and analyse data in order to investigate what teachers do as they teach mathematics and what mathematical knowledge and skills teachers need to hold in order to be able to teach mathematics effectively. For answering these questions initially the team observed mathematics lessons in a third grade public school during 1989-1990 (Ball et al, 2007). The lessons observed were videotaped and audio taped. All the data were transcribed and qualitatively analysed by the research team who aimed to develop a theory of mathematical knowledge that is needed in teaching mathematics. As a complement to the videotaped data, the researchers also collected copies of students' work in the class, students' written work in their homework and quizzes, and teachers' lesson plans.

Moreover, in addition to the set of data collected during 1989-1990 the research team over the last ten years worked on collecting data from observations of mathematics lessons. These data also include videotapes of classroom teaching, samples of students' written work, teachers' lesson plans, and curriculum materials that were used by the teachers in their lessons. `

All the data collected through observations of mathematics lessons were qualitative analysed. From this analysis the team developed a practice-based description of what they call "mathematical knowledge for teaching" (Hill and Ball, 2004). Mathematical knowledge for teaching refers to knowledge that teachers need to hold in order to teach mathematics effectively. Based on these first findings from the observations the research team suggested that Shulman's (1986) content knowledge of mathematics can be divided into two categories (Ball et al, 2005). The first category is called common content knowledge and refers to the mathematical knowledge used in settings other than teaching and includes teachers' ability to solve mathematical problems correctly. The second category is that of specialized content knowledge and refers to knowledge that only teachers need to hold in their work (Ball et al, 2007)

Summarising the above the initial results from the lessons observed suggested that there is mathematical knowledge for teaching that is specialised for the work that teachers do. The research team decided that it was necessary to find ways to check this hypothesis by developing measures of such knowledge. Without this empirical testing the idea of specialised content knowledge remains hypothetical and plays limited role in improving mathematics teaching and learning (Ball et al, 2005). The design of such measures was the focus of the LMT project.

2.3 Learning Mathematics for Teaching Project

The LMT project aims to investigate the mathematical knowledge needed by teachers in order to teach mathematics effectively. This is done by developing measures of mathematical knowledge for teaching. The research team recognised that a large sample of mathematics teachers and many items would be needed in order to test empirically the hypotheses of their qualitative studies. The large sample size would help to control other factors that might relate to students' learning and to effective mathematics teaching. Therefore, they decided to develop multiple-choice mathematical items that can be answered by large number of participants and permit broad coverage of the mathematical topics being the focus of the study, due to participants' ability to respond to many items. (Ball et al, 2005)

The first step in designing the items was to decide what mathematical topics to be measured. The decision was to focus on numbers and operations, two areas that were central to elementary school mathematics in the US and are extremely important for students learning. Furthermore, the team decided to cover the domains of algebra, patterns functions and geometry as these represented a new stand in K-6 curriculum in the US.

Once the mathematical areas to be covered were specified, a team of experts was asked to design a number of items. In the designing team were involved mathematics educators, mathematicians, professional developers, members of the MTLT and LMT projects and classroom teachers. For the design of the items the design team was given a numbers of guidelines. First, all the items must have multiple-choice format. Second, the items should describe situations that teachers might face in real classroom situations. Third, the items should measure mathematical knowledge for teaching as this was conceptualised in the MTLT project and capture both common knowledge of mathematics and specialised knowledge of mathematics. Finally, for the development of the items the design team was informed by the qualitative studies done as a part of the MTLT project, by existing literature on teaching and learning mathematics (e.g. Ball 1990, Ma, 1999), by the analysis of curriculum materials and examples of students' work. Figures 1 and 2 are examples of the items developed by the team (Ball et al 2005; Hill and Ball, 2004). Figure 1 represents an item that was designed to measure common content knowledge. Although, this item represents a real classroom situation, it requires only understanding that every educated adult might hold. It does not require any special knowledge that only teachers need to hold. Figure 2 represents an item that was designed to measure specialised content knowledge. In this item respondents must know to multiply 35 x 25 using the standard algorithm but also be able to recognise students' errors and be able to identify their source. An adult should know how to multiply 35 x 25 but the ability to recognise students' errors is knowledge uniquely needed for teachers to be effective. The two examples presented below, as well as all the items designed (currently the team has developed over 250 items), before taking their final format, were reviewed, criticized, revised and pilot-tested.

Ms. Domiguez was working with a new textbook and she noticed that it gave more attention to the number 0 than the old book. She came across a page that asked students to determine if a few statements about 0 were true or false. Intrigued, she show them to her sister who is also a teacher, and ask her what she thought.
Which statement(s) should the sister select as being true? (Mark YES, NO, or I'M NOT SURE for each item below

	Yes	**No**	**I'm not sure**
a) 0 is an even number.	1	2	3
b) 0 is not really a number. It is a placeholder in writing big numbers.	1	2	3
c) The number 8 can be written as 008.	1	2	3

Figure 1: Item measuring common content knowledge (Hill and Ball, 2004, p.346)

Imagine that you are working with your class on multiplying large numbers. Among your students' papers, you notice that some have displayed their work in the following ways:

Student A	**Student B**	**Student C**
35	35	35
x 25	x 25	x 25
125	125	25
+ 75	+ 700	150
875	875	100
		+ 600
		875

Which of these students would you judge to be using a method that could be used to multiply any two whole numbers?

	Method would work for all whole numbers	Method would **NOT** work for all whole numbers	I'm not sure
a) Method A	1	2	3
b) Method B	1	2	3
c) Method C	1	2	3

Figure 2: Item measuring specialised content knowledge (Hill and Ball, 2004, p.346)

The claim that the items designed can measure mathematical knowledge for teaching and give good predictions of classroom effectiveness required more empirical evidence. Therefore, it was important for the team to empirically check whether high performance on the items can predict effective teaching or better students learning. Over the last years the Michigan's mathematics education research team had the chance to be involved in various projects that provide them with the opportunity to pilot their items on a large scale.

For example the team designed an instrument based on their items in order to evaluate California's Mathematical Professional Development Institute (CMPDI). The team was asked to investigate whether teachers developed mathematical knowledge through the professional development program they participated and if they did, what features of professional development contributed to knowledge development (Hill and Ball, 2004). To answers this question a pre-test and a post-test were developed to measure teachers' knowledge. The analysis of the data showed that teachers' participation in the professional development program improved their performance on the test items. The team's involvement in the CMPDI evaluation was a good opportunity to pilot their items on a large scale, and check their reliability and validity. Validity checks included:

- comparison of teachers' score to the items measuring their mathematical knowledge for teaching and the use of their mathematical knowledge in real classroom situations (this work continues up today)

- interviews in which teachers were asked to explain their thinking in answering the multiple-choice items. The analysis of the interviews showed that teachers' answers to the items indeed revealed their thinking.

- comparison of the item topics with the NCTM standards and California's Mathematics framework.

In another project Hill, Rowan and Ball (2005) set out to investigate whether teachers' scores on their items can predict that students learn more or better. In order to answer this question Hill, Rowan and Ball designed an instrument based on the multiple-choice items developed for measuring mathematical knowledge for teaching. Their analysis showed that teachers' performance on the items significantly predicted gains in students' achievement, even though aspects such as students' socioeconomical background, students' absence rate and teachers' experience were controlled.

The two examples of research projects described above, as well as the use of the items in other studies, revealed that the items developed for measuring mathematical knowledge for teaching can give good predictions of classroom effectiveness and students' learning. Furthermore, analytic techniques, such as factor analysis were used to check the hypothesis about the structure of mathematical knowledge for teaching. The quantitative analysis helped the team to test and refine categories of mathematical knowledge of teaching. Although, this analyses are still ongoing they suggest that mathematical knowledge has many dimensions. In the section below I describe the model of mathematical knowledge for teaching that the team proposes.

THE STRUCTURE OF 'MATHEMATICAL KNOWLEDGE FOR TEACHING'

Based on the analysis of both quantitative and qualitative data over the last fifteen years the Michigan's mathematics education research team proposes a knowledge model according to which mathematical knowledge for teaching can be divided into four categories. The model they proposes builds on Shulman's (1986,1987) work on teachers' knowledge and suggests that Shulman's contents knowledge can be divided in two categories, the common content knowledge and the specialised content knowledge. In addition to this, they suggest that PCK as this is conceptualised by Shulman can be divided into Knowledge of Content and Students (KCS) and Knowledge of Content and Teaching (KCT) (Ball et al, 2007).

As mentioned above common content knowledge refers to mathematical knowledge and skills that are used in any setting and not necessarily in the setting of teaching and includes individual's ability to calculate an answer and to solve mathematical problems correctly. Specialised content knowledge is the knowledge that is used in classroom settings and is needed by teachers in order to teach effectively. (Ball et al, 2007).

KCS is "knowledge that combines knowledge about students and knowing about mathematics" (Ball et al 2007, p.36). This in turn means that teachers must be able to anticipate students' difficulties and obstacles, to hear and respond appropriately to students' incomplete thinking and finally to choose appropriate examples and representations while teaching. In addition teachers must show awareness of students' conceptions and misconceptions about a mathematical topic.

Finally the KCT is "knowledge that combines knowledge about mathematics and knowledge about teaching" (Ball et al, 2007, p.38). It refers to teachers decisions on sequencing of activities and exercises, to their awareness of possible advantages and disadvantages of representations used while they teach and finally, to their decisions to pause a classroom discussion for more clarifications, or their decision to use a student's opinion to make a mathematical remark (Ball et al, 2007)

KCS and KTC as are conceptualised by the team are the same as two central dimension of PCK as this is defined by Shulman (1986). These are first teachers' awareness of their students' conceptions and misconceptions and second the use of representations, examples that teachers use in order to make subject matter comprehensible to students

CONCLUSION

Over the last fifteen years the Michigan mathematics education research team devised instruments to measure both teachers' SMK and PCK and showed that these give good predictions of classroom effectiveness and students learning. These measures helped the team to develop a practice-based theory of content knowledge building on Shulman's notion of Pedagogical Content Knowledge (PCK) and propose new categories of teachers' mathematical knowledge for teaching. This practice-based theory might be very helpful for further research. For example when investigating the relationship between teachers' knowledge and students' achievement the researchers can use the model proposed and identify which categories of mathematical knowledge for teaching are the greatest predictors of students' achievement or effective mathematical thinking (Ball et al, 2007). Also, a clearer understanding of what teachers knowledge includes can help policy makers to develop more effective curriculum for teachers' training programmes.

REFERENCES

Ball, D. L. (1990) 'Prospective elementary and secondary teachers' understanding of division.' *Journal for Research in Mathematics Education* 21(2), 132-144.

Ball, D. L., Hill, H.C, and Bass, H. (2005) 'Who knows mathematics well enough to teach third grade, and how can we decide?' *American Educator*. http://www.aft.org/pubsreports/american_educator/issues/fall2005/BallF05.pdf

Ball, D.L., Thames, M.H., and Phelps, G. (2007). 'Content knowledge for teaching: What makes it special?' Manuscript submitted for publication. http://www-personal.umich.edu/~dball/papers/index.html

Hill, H.C., and Ball, D. L. (2004) ‘Learning mathematics for teaching: Results from California’s mathematics professional development institutes.’ *Journal for Research in Mathematics Education* 35 (5), 330-351.

Hill, H.C., Rowan, B., and Ball, D. (2005) ‘Effects of teachers' mathematical knowledge for teaching on student achievement.’ *American Educational Research Journal*, 42 (2), 371- 406.

Ma, L. (1999) *Knowing and Teaching Elementary Mathematics: Teachers’ Understanding of Fundamental Mathematics in China and the United States.* London: Lawrence Erlbaum Associates.

Shulman, L. S. (1986) ‘Those who understand, knowledge growth in teaching.’ *Educational Researcher* 15(2), 4-14.

Shulman, L. S. (1987) ‘Knowledge and Teaching: Foundations for the new reform’. *Harvard Educational Review 57* (1), 1-22.