What kinds of knowledge help teachers to become effective teachers of mathematics? What kinds of choices do teachers have? - a comparative perspective

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"The single factor which seems to have the greatest power to carry forward our understanding of the teachers' role is the phenomenon of teachers' knowledge." (Elbaz, 1983, p.45)

In recent years the question dealing with teacher knowledge has received an increasing amount of attention by researchers. Researchers have investigated the professional knowledge of teachers from different angles. It is accepted that what teachers know is one of the most important influences on what happens in classrooms. However, it has not been possible to establish a strong relationship between teachers' mathematical knowledge and their students' achievement. The persistent failure to show strong and definitive relations between teachers' mathematical knowledge and their effectiveness does not imply that mathematical knowledge makes no difference in teaching (Ball, 2001). It has been established, however, that the conceptual tools that teachers posses in order to deal with their work depend to a large extend on the cultural (and structural) traditions of the educational environment in which they are working (Stigler & Hiebert, 1999; Hiebert et al, 2003; Pepin, 1999; Andrews & Hatch, 2000). However, there is no consensus on what teachers need to know in order to ensure that pupil learning is taking place.

In terms of mathematics teacher knowledge there is evidence that insufficient and poor mathematical knowledge has a negative impact on teaching, and researchers argue about the nature of that knowledge. On the other hand, work at King's College London has apparently found no link between teachers' subject knowledge, measured in terms of academic qualifications, and effective primary school teaching (Askew et al, 1997). Many researchers show that mathematics as knowledge to practice mathematics is distinct from that for teaching mathematics. Ball (2003) argues that mathematics-for-teaching is unlikely to be neither 'more of' or 'to a greater depth than' the knowledge expected of students, but that it is qualitatively different.

"…knowledge for teaching mathematics is different from the mathematical knowledge needed for other mathematically-intensive occupations and professions. The mathematical problems and challenges of teaching are not the same as those faced by engineers, nurses, physicists, or astronauts. Interpreting someone else's error, representing ideas in multiple forms, developing alternative explanations, choosing a usable definition—these are all examples of the problems that teachers must solve. These are genuine mathematical problems central to the work of teaching." (Ball, 2003, p.6/7)

Ball and Bass (2003) argue for mathematics-for-teaching to be/become a distinct branch of mathematics, and there is a growing area of research concerned with this. They argue that mathematics knowledge for teaching is not a watered down version of 'real' (university) mathematics, but a demanding area of mathematical work.

" …the mathematical knowledge needed for teaching must be usable for ***those*** mathematical problems. Mathematical knowledge for teaching must be serviceable for the mathematical work that teaching entails, from offering clear explanations, to posing good problems to students, to mapping across alternative models, to examining instructional materials with a keen and critical mathematical eye, to modifying or correcting inaccurate or incorrect expositions. The mathematical knowledge needed for teaching, even at the elementary level, is not a watered-down version of "real" mathematics. Teaching mathematics is a serious and demanding arena of mathematical work." (Ball, 2003, p.7)

There appears to be a lack of agreement among experts, policymakers, and reformers about competing views concerning the importance of subject-matter and pedagogy; and disagreement over what teachers learn from experience, also in particular contexts, among others. There is, however, an agreement that knowledge is a crucial issue in the quality of mathematics teachers (Ball, 2001).

In the following I outline and briefly discuss salient theoretical issues associated with the conceptualisation of teacher mathematics knowledge and beliefs; and the situated (and distributed) character of mathematics teacher knowledge.

First, since the 1980s and under the influence of Shulman (1987) teachers' professional knowledge has been conceptualised as subject-matter content knowledge, pedagogical content knowledge (PCK) and general pedagogical knowledge. Although there is much agreement on PCK as an integral part of teacher knowledge, it appears difficult to conceptualise and measure it (e.g. in TEDS-M). The difference about the nature of PCK raises a number of conceptual challenges, reflected in differences in positions taken by experts. For example, what is 'practical knowledge'? Is this knowledge idiosyncratic in nature, and acquired in teaching in particular situations and settings? If this was the case, this would have implications for generalisability and transfer from one context to another, and could not easily be assessed. Different epistemological traditions and stances provide different answers (e.g. Ball, 2001; De Corte & Verschaffel, 1996; Munby et al, 2001). Interestingly, Ma (1999), and also Pepin (in this seminar), demonstrate that there can be important differences from country to country, underpinned by educational and cultural traditions, in the nature of knowledge for teaching mathematics.

Furthermore, there has been much debate about what means to understand in mathematics, what constitutes knowledge in mathematical learning. A classification commonly used is the distinction between procedural and conceptual knowledge, or between procedural and relational knowledge (Skemp, 1976). Others (e.g. Brodie, 2004) argue this is too limited and that a more interactive conceptualisation is needed. For example, Kilpatrick et al (2001) propose five interrelated strands of 'mathematical proficiency': conceptual understanding; procedural fluency; strategic competence; adaptive reasoning; and productive disposition. As another example, Ma (1999) analyses the attributes of teacher understanding and describes the nature and development of the "profound understanding of fundamental mathematics" (PUFM) that elementary teachers need to become accomplished mathematics teachers. Interestingly, she claims that such teaching knowledge is more common in China than in the US, even though Chinese teachers have less formal education than their US counterparts.

What becomes clear here is that teacher knowledge cannot be separated from "the subject matter being investigated, from how that subject matter can be represented for learners, from what we know about students' thinking in particular domains, or from teacher beliefs" (Fennema & Franke, 1992, p.161).

In addition to content knowledge and pedagogical knowledge as conventionally construed, there is also the aspect of teacher beliefs, meta-cognitive processes ("thinking about thinking", e.g. Sternberg 1992) and how teachers think about mathematics (see also Sfard, 1998, comparison of two metaphors of learning). Since the early work of Thompson (1984) there is an agreement on the importance of teachers' beliefs in shaping teacher practice and professional development. When studying teaching and learning, Ernest (1989) identifies and describes the following views: Problem solving; Platonist; Instrumentalist. Dionee (1984) suggests that mathematics can be seen as one of (or a combination of) three basic components (*traditional, formalist and constructive* perspective), and Toerner & Grigutsch (1994) refer to the *toolbox* aspect, the *system* aspect and the *process* aspect. De Corte et al (2000) offer useful classifications of sub domains involved (they relate it to students, but these are also useful and relevant for teachers): beliefs about mathematics education; about the self in relation to mathematics; about the social context of mathematical learning and problem-solving.

Moreover, there is much agreement that teaching represents what has come to be called situated knowledge, knowledge of and adapted to particular contexts (Putnam & Borko, 2000). To measure this, it requires attention to the varied classroom settings in which teachers practice, and also to teachers' own schooling ("apprenticeship of learning", Lortie, 1976) and their teacher education (courses) at university and in schools. The knowledge developed and shaped by each of these contexts contrasts with what has been called general knowledge (that is applicable across situations and settings) and moreover with theoretical knowledge (that is rooted in basic ideas and concepts). Thus, we may ask, is it sufficient to measure general/theoretical knowledge without regard for its situated character? Ball (2001) is clear that "what matters ultimately is not only what courses teachers have taken or even what they know, but also whether and how teachers are able to use mathematical knowledge in the course of their work."

From this it is clear that mathematical knowledge in teaching is a "large, integrated, functioning system with each part difficult to isolate" (Fennema & Franke, 1992). The enclosed map (see fig. 1) represents some of the relationships within the 'web' of teacher knowledge. One consistent view across all conceptions of teacher knowledge is that it is continually changing and developing - it is not static. It grows through interaction with the subject, with students and through professional experiences, among others - it is a process, and this process takes time (Fennema & Franke, 1992). This change element has to be born in mind when studying teacher knowledge.

Over the past decades researchers and policy makers have discussed, in a lively fashion, whether cross-national (and cross-cultural) studies may inform educational practice in any one country (e.g. Berliner & Biddle, 1995; Stigler & Hiebert, 1998; Le Tendre et al, 2001). In this proposed research we take a slightly different stance. Instead of asking "should we be more like this?", we would like to ask the following questions:

- What 'constellations' of knowledge helps teachers to become 'effective' teachers of mathematics- what are the factors and what are the relationships between the different 'kinds' of knowledge in different cultural-historical and national contexts?
- If these factors, and the relationship between those factors, change, what might it be like?
- What can be changed, what kinds of choices do teachers have, in terms of change? What are 'key levers' of change?

In order to examine the different 'constellations' of knowledge and their potential change, we believe that a comparative perspective helps

1. to identify the different 'constellations' (factors and relationships between factors) more clearly;
2. to better understand the possibilities for impact of cultural change of teacher knowledge.

In the past, and in international studies such as TIMSS (and the recent TEDS-M), researchers have commonly focussed on comparisons between East Asian and Western cultures. We believe that this not only backgrounds the 'nuances' that are evident within the European systems (see Pepin 1999), but also is less helpful to understand which choices we have in 'our sphere' - and we thus propose a comparative study of teacher knowledge in selected European countries.

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