THE KNOWLEDGE QUARTET: A MEANS OF DEVELOPING AND DEEPENING MATHEMATICAL KNOWLEDGE IN TEACHING?

Fay Turner and Tim Rowland, University of Cambridge

In this paper, a practice-based framework for the identification and discussion of prospective elementary school teachers' mathematics content knowledge is described. This framework - 'the Knowledge Quartet' - emerged from intensive scrutiny of 24 videotaped lessons, taught by novice teachers. Application of the Knowledge Quartet in lesson observation and teacher education is illustrated with reference to a particular lesson taught by one trainee teacher.

The text then proceeds to locate the first author's current longitudinal study of beginning primary school teachers in the context of some of the themes arising from previous MKiT seminars. A case is made for the claim that mathematical content knowledge for teaching is most usefully identified in the act of teaching, and developed through reflection on teaching. The study reported in this second paper of the paper adopts an approach to developing mathematics teaching based on reflection, using the Knowledge Quartet, framed by socio-cultural theories of learning.

INTRODUCTION

Organisation

This paper is written for the fifth Nuffield MKiT Seminar, against the backdrop of the earlier meetings. The paper is in two parts. The first is a succinct account of a theory of mathematical knowledge in teaching developed some five years ago by a team which included the two authors, at Cambridge. The 'Knowledge Quartet' is a 'theory' in the sense that it proposes a way of thinking about mathematics teaching in the usual institutional settings (lessons), with a focus on the disciplinary content (mathematics) of the lesson. Readers familiar with the Knowledge Quartet could proceed directly to the second part of the paper, in which Fay Turner discusses a longitudinal project which investigates use of the Knowledge Quartet as an analytical 'tool' to support the development of early-career primary school teachers' mathematics content knowledge (SMK *and* PCK) .

PART I: THE KNOWLEDGE QUARTET

Rationale

In the UK, most prospective ('trainee') teachers follow a one-year, postgraduate course leading to a Postgraduate Certificate in Education (PGCE) in a university education department. About half of the PGCE year is spent working in schools as an 'intern', under the guidance of a school-based mentor. Research shows that feedback on observation of lessons taught by student teachers typically focuses heavily on organisational features of the lesson, with very little attention to the subject-matter being taught (e.g. Strong and Baron, 2004). The purpose of the research reported here was to develop an empirically-based conceptual framework for the discussion of the role of trainees' mathematics SMK and PCK, in the context of lessons taught on their school-based placements. Such a framework would need to capture a number of important ideas and factors about content knowledge within a small number of conceptual categories, with a set of easily-remembered labels for those categories.

Recent studies of Deborah Ball and her colleagues at the University of Michigan (for a summary, see Petrou, 2007) have been directed towards a "practice-based theory of knowledge for teaching" (Ball and Bass, 2003). The same description could be applied to our own study, but while parallels can be drawn between the methods and some of the outcomes, the two theories look very different. In particular, the theory that emerges from the Michigan studies attempts to unravel and clarify the notions of SMK and PCK. As a consequence, Shulman's SMK is separated into 'common content knowledge' and 'specialized content knowledge', while his 'pedagogical content knowledge' is divided into 'knowledge of content and students' and 'knowledge of content and teaching'. In our own theory, as will become apparent later, the distinction between different kinds of mathematical knowledge is of lesser significance than the classification of the situations in which mathematical knowledge surfaces in teaching. In this sense, the two theories may each have useful perspectives to offer to the other.

Method

12 trainees on a one-year primary PGCE course were chosen for observation during their final school placement. Two mathematics lessons taught by each of these trainees were observed and videotaped, i.e. 24 lessons in total. We took a grounded approach to the data for the purpose of generating theory (Glaser and Strauss, 1967). In particular, we identified aspects of trainees' actions in the classroom that seemed to be significant in the limited sense that they could be construed to be informed by their mathematics SMK or PCK. These actions were located in particular moments or episodes in the tapes. This inductive process generated a set of 18 codes. Next, we revisited each lesson in turn and, after intensive study of the tapes, wrote an 'analytical account' of the lesson. In these accounts, significant moments and episodes were identified and coded, with appropriate justification and analysis concerning the role of the trainee's content knowledge in the identified passages, with links to relevant literature.

Our catalogue of 18 codes[1] presented us with the following problem. We intended to offer our findings to colleagues for their use, as a framework for reviewing trainees' mathematics content knowledge from evidence gained from classroom observations of teaching. We anticipated, however, that 18 codes is too many to be useful for a one-off observation. Our resolution of this dilemma was to group them into four broad, super-ordinate categories, or 'units', which we term 'the Knowledge Quartet'[2]. A detailed methodological account is given in Rowland (in press).

Conceptualising the Knowledge Quartet

We have named the four units of the Knowledge Quartet as follows: foundation; transformation; connection; contingency. Each unit is composed of a small number of cognate subcategories. For example, the third of these, *connection,* is a synthesis of four of the original 18 codes, namely: *making connections; decisions about sequencing; anticipation of complexity,* and *recognition of conceptual appropriateness.* Our scrutiny of the data suggests that the Quartet is comprehensive as a tool for thinking about the ways that subject knowledge comes into play in the classroom. However, it will become apparent that many moments or episodes within a lesson can be understood in terms of two or more of the four units; for example, a **contingent** response to a pupil's suggestion might helpfully **connect** with ideas considered earlier. Furthermore, it could be argued that the application of subject knowledge in the classroom *always* rests on **foundational** knowledge. Drawing on the extensive range of data from the 24 lessons, we offer here a brief conceptualisation of each unit of the Knowledge Quartet. A more detailed account is given in Rowland, Huckstep and Thwaites (2005).

The first category, *foundation*, consists of trainees' knowledge, beliefs and understanding acquired 'in the academy', in preparation (intentionally or otherwise) for their role in the classroom. The key components of this theoretical background are: knowledge and understanding of mathematics *per se* and knowledge of significant tracts of the literature on the teaching and learning of mathematics, together with beliefs concerning the nature of mathematical knowledge, the purposes of mathematics education, and the conditions under which pupils will best learn mathematics. The second category, *transformation*, concerns knowledge-in-action as demonstrated both in planning to teach and in the act

---

[1] A 19th code emerged much later (Rowland, 2007; in press) as a result of theoretical sampling.

[2] Specifically, the codes contributing to each of the four units are as follows:

FOUNDATION: adheres to textbook; awareness of purpose; concentration on procedures; identifying errors; overt subject knowledge ; theoretical underpinning; use of terminology.

TRANSFORMATION: choice of examples; choice of representation; demonstration.

CONNECTION: anticipation of complexity; decisions about sequencing; recognition of conceptual appropriateness; making connections between concepts; making connections between procedures.

CONTINGENCY: deviation from agenda; responding to children's ideas; use of opportunities; teacher realisation.

of teaching itself. As Shulman indicates, the presentation of ideas to learners entails their re-presentation (our hyphen) in the form of analogies, illustrations, examples, explanations and demonstrations (Shulman, 1986, p. 9). Of particular importance is the trainees' choice and use of examples presented to pupils in connection with the acquisition of concepts and procedures, and also in the cause of mathematical enquiry. The third category, *connection*, binds together certain choices and decisions that are made for the more or less discrete parts of mathematical content. In her discussion of 'profound understanding of fundamental mathematics', Liping Ma cites Duckworth's observation that intellectual 'depth' and 'breadth' "is a matter of making connections" (Ma, 1999, p. 121). Our conception of this coherence includes the sequencing of material for instruction, and an awareness of the relative cognitive demands of different topics and tasks. Our final category, *contingency*, is witnessed in classroom events that were not anticipated or planned for. In commonplace language it is the ability to 'think on one's feet'. In particular, the readiness to respond to children's ideas and a consequent preparedness, when appropriate, to deviate from an agenda set out when the lesson was prepared.

Applying the Knowledge Quartet

We now proceed to a brief illustration how the Knowledge Quartet is being applied, by reference to one of the 24 videotaped lessons. The trainee in question, Laura, was teaching a Year 5 (pupil age 9-10) class about written multiplication methods. The key focus of this lesson was on teaching column multiplication of whole numbers, specifically multiplying a two-digit number by a single digit number. Laura reminded the class that they have recently been working on multiplication using the 'grid' method (see below). Simon was invited to the whiteboard to demonstrate the method for 9x37. He wrote:

x	30	7		
9	270	63	=	333

Laura then said that they would be learning "another way". She proceeded to write the calculation for 9x37 on the whiteboard in a conventional but elaborated column format, explaining as she goes along:

	37
	x 9
30x9	270
7x9	63
	333
	1

Laura performs the sum 270+63 by column addition *from the right*, 'carrying' the 1 (from 7+6=13) from the tens into the hundreds column. She writes the headings h, t, u above the three columns.

Next, Laura showed how to "set out" 49x8 in the new format, followed by the first question (19x4) of the exercises to follow. There was 24 minutes' work on exercises that Laura had displayed on the wall.

In a concluding eight-minute plenary, Laura asked one boy, Sean, to demonstrate the new method with the example 27x9. Sean got into difficulty; he wrote 27 and x9 in the first two rows as expected, but then wrote 20x7 and 2x9 to the left in the rows below.

We now select from Laura's lesson a number of moments, episodes and issues to show how they might be perceived through the lens of the Knowledge Quartet. Some possibilities for discussion with the trainee, and for subsequent reflection, are flagged below thus: **Discussion point**. We emphasise that the process of *selection* in the commentary which follows has been extreme.

Foundation

First, Laura's professional knowledge – a case of what the Michigan team would surely call 'specialized content knowledge' - underpins her recognition that there is more than one possible written algorithm for whole number multiplication. We conceptualise this within the domain of fundamental knowledge, being the *foundation* that supports and significantly determines her intentions or actions. Laura's learning objective seems to be taken from the National Numeracy Strategy (NNS) *Framework* (DfEE, 1999) teaching programme for Year 4:

> Approximate first. Use *informal* pencil and paper methods to support, record or *explain* multiplication. *Develop* and *refine* written methods for TUxU (p. 3/18, emphasis added)

These objectives are clarified by examples later in the *Framework*; these contrast (A) *informal* written methods - the grid, as demonstrated by Simon - with (B) *standard* written methods - the column layout, as demonstrated by Laura in her introduction. It is perhaps not surprising that Laura does not *question* the necessity to teach the standard column format to pupils who already have an effective, meaningful algorithm at their disposal. A best-selling handbook for trainee primary teachers, written by a respected teacher educator, explains why the standard algorithm works, but forcibly advocates the adequacy and pedagogical preference of grid-type methods with primary pupils (Haylock, 2006).

> **Discussion point**: where does Laura stand on this debate, and how did this contribute to her approach in this lesson?

At this stage of her career in teaching, Laura gives the impression that she is passing on her own practices and her own forms of knowledge. Her main resource seems to be her own experience of using this algorithm.

Transformation

Laura's choice of demonstration examples in her introduction to column multiplication merits some consideration and comment. Her first example is 37×9; she then goes on to work through 49×8 and 19×4. Now, the NNS emphasises the importance of *mental* methods, where possible, and also the importance of choosing the most suitable strategy for any particular calculation. 49×8 and 19×4 can all be efficiently performed by rounding up, multiplication and compensation e.g. 49×8 = (50×8)-8.

Her choice of exercises - the practice examples - also invites some comment. The sequence is: 19x4, 27x9, 42x4, 23x6, 37x5, 54x4, 63x7, 93x6, with 99x9, 88x3, 76x8, 62x43, 55x92, 42x15 as 'extension' exercises (although no child actually attempts these in the lesson). Our earlier remark about the suitability of the column algorithm relative to alternative mental strategies applies to several of these, 99x9 being a notable example.

Furthermore, the *sequence* of exercises might be expected to be designed to present the pupils with increasing challenge as they progress though them.

> **Discussion point**: on what grounds did Laura choose these particular examples and exercises? What considerations might contribute to the choice?

Connection

Perhaps the most immediate connection to be established in this lesson is that between the grid method and the column algorithm. Laura seems to have this connection in mind as she introduces the main activity. She reminds them that they have used the grid method, and says that she will show them a "slightly different way of writing it down", although after the first example is completed Laura says that they are learning a "different way to work it out". She says that the answer would be the same whichever way they did it "because it's the same sum". Of course, that presupposes that both methods are valid, but does not clarify the connection between them: that the same processes and principles - partition, distributivity and addition - are present in both methods. The fact that Laura includes demonstrations of 37x9 by both methods does help to establish the connection, but the effort to sustain the connection is not maintained, and no reference to the grid method is made in her second demonstration example, 49x8.

> **Discussion point**: Laura seems to be trying to make a connection between the grid method and the column method. How did she intend to make that connection? To what extent did she think she was successful?

Contingency

Sean's faulty attempt (mentioned above) to calculate 27x9 on the whiteboard appears to have taken Laura by surprise. There are several 'bugs' in his application of the procedure. The partition of 27 into

20 and 2 is faulty, and the multiplicand is first 9, then 7. This would seems to be a case where Sean might be encouraged to reconsider what he has written by asking him some well-chosen questions. One such question might ask how he would do it by the grid method. She asks the class? “Is that the way to do it? Would everyone do it that way?”. Leroy demonstrates the algorithm correctly, but there is no diagnosis of where Sean went wrong, or why.

> **Discussion point**: what might be the reason for Sean’s error? In what ways could this have been addressed in the lesson, or subsequently?

Concluding (Pt I) remark

Research originally fuelled by curiosity about teacher knowledge and classroom practices led to the development of the Knowledge Quartet, a manageable framework within which to observe, analyse and discuss mathematics teaching. The framework is in use in teacher education programmes in Cambridge and elsewhere. Those who use it need to be acquainted with the details of its conceptualisation. Initial indications are that this development has been well-received by teacher-mentors, who appreciate the specific focus on mathematics content and pedagogy.

PART II: USING THE Knowledge Quartet TO DEVELOP AND DEEPEN MkIT

Conceptualising and identifying mathematical content knowledge for teaching

An aim which has been implicit throughout the MKiT seminar series, and is explicit for seminar 5, is that of determining how teachers’ mathematical content knowledge for teaching might be identified and developed. A first step towards such an aim was to conceptualise what is meant by ‘mathematical knowledge in teaching’. Maria Goulding cited the seminal work of Lee Shulman (1986) as a starting point towards defining such knowledge. Shulman’s categories of teacher knowledge have given a useful basis for many contributions to this debate.

Seamus Hegarty (2000) however, argued that ways of categorising teachers’ knowledge do not themselves explain how teacher knowledge affects classroom behaviour and so does not help in understanding effective teaching. Hegarty argues that the effects of all the different areas of teacher knowledge can only be understood within the contexts of dynamic teaching situations. He presented a model in which the teacher has a number of incomplete sets of relevant insights, elements of which come together in instances of teaching to form a new insight specific to that situation. According to this model, teacher knowledge is firmly situated within the act of teaching and must be observed through such acts.

Shulman would not seem to disagree with this idea. In addition to developing categories of teacher knowledge, he proposed that knowledge for teaching exists in three different forms. The first of these he terms *propositional knowledge* and is the theoretical knowledge held by teachers. The second form of knowledge, *case knowledge* develops through experience of teaching situations and finally, *strategic knowledge* is that which refers to the way teachers act in the moment while teaching. Knowledge in this third form would seem to be similar to that proposed by Hegarty. Propositional knowledge might be measured without observations of teaching. Case study knowledge might also be measured away from the classroom, but could also be observed in the act of teaching. Strategic knowledge can, by definition, only be observed in teaching.

In a briefing paper for the third MKiT meeting, Marilena Petrou summarised recent work of Deborah Ball and her associates in the US. This included a report on the development of a questionnaire for measuring ‘professional mathematical knowledge’ which, though based on observations of teaching, may also be used independently of observations (Ball, Hill and Bass, 2005). This instrument is claimed to measure both SMK and PCK through presenting teachers with teaching situations and asking them to answer multiple choice questions about them. It can be argued however, that though this instrument can measure propositional and case knowledge, it cannot measure the strategic knowledge which most visibly affects the learning of children.

Developing teachers' mathematical content knowledge

This seminar series has not only aimed to define what knowledge is needed for mathematics teaching and how such knowledge might be identified. It also aims to understand how such knowledge might be developed in teachers. Jeremy Hogden's and Julian Williams' papers both took the socio-cultural perspective that any 'judgements' about teachers' mathematical content knowledge for teaching must be made in the context of their teaching, since knowledge is located in social practice (Lave, 1988; Greeno, 1998; Putman and Borko, 2000). Recognising the situated nature of knowledge for teaching has clear implications for the development of such knowledge. If knowledge is located in a socio-cultural context, then development of such knowledge must also take place in this context.

There are many proponents of socio-cultural theory models of teacher learning (Lave, 1988; Lave and Wenger, 1991; Cobb, Yackel and Wood, 1991; Wenger, 1998; Lerman, 2001). In such models, teachers' learning is seen as situated in classrooms and grounded in interactions with students (Cobb, Yackel and Wood, 1991). Furthermore, learning is not seen to happen in isolation, but within professional communities of practice (Wenger, 1998). Cobb and McClain (2001) explained how their thinking had progressed from seeing teacher learning as chiefly situated in their classrooms and grounded in their interactions with students to recognising the importance of interactions with other professionals.

Calderhead and Shorrock (1997) proposed a socio-cultural model of professional development which they termed *enculturation* or *socialization into the professional culture*. They also proposed a more individualistic model of professional development which they termed *professional development through reflection*. Calderhead and Shorrock suggested that no single model on its own will give the whole picture, but that they each offer some help in understanding development, and are all interrelated. The model of professional development through reflection is often referred to in initial teacher education. Donald Schön (1983) identified two forms of reflection; *reflection-in-action* – the process of monitoring and adapting behaviour in context, and *reflection-on-action* – after the event evaluation. Reflection may be seen as an individual activity performed by the teacher in isolation. However this is not necessarily the case and it can be argued that reflection is more effective in developing practice when carried out within a 'community of practice'.

Barbara Jaworski (1998) demonstrated development in the mathematics teaching of teachers through reflection, when working on their own action research projects. In her later work (Jaworski, 2001) she emphasised the importance of communities of practice, in addition to reflective activity, in such development as they "provide opportunities for sharing experiences, synthesising from and explaining outcomes of research and developing critical frameworks related to practice." (p. 298).

Lerman (2001) was also adamant in his view that reflection alone does not lead to learning but must involve others.

> Reflection per se, does not give us enough to serve as a process of learning. This is not to say that we don't reflect, only that for reflection to say something about how people learn involves others in one way or another. Reflective practice takes place in communities of practice, as groups of teachers in a school, teachers attending in-service courses, or other situations, and learning can be seen as increasing participation in that practice. (p. 41)

But what if the practice into which teachers are being 'enculturated' is not that which might be considered 'good' practice? Fennema and Franke (1992) suggest that teachers subject matter knowledge (SMK), their pedagogical content knowledge (PCK) and their knowledge of students is 'firmly situated' within their own school experience. Though this knowledge may be developed through teaching experiences, pedagogical content knowledge can only be developed in a positive direction if situations in which teachers learn, reflect received good practice. Brown, McNamara, Jones and Hanley (1999) also suggested that development facilitated by integration into communities of practice may not lead to mathematics teaching that is consistent with contemporary views of effective teaching (Cockcroft, 1982; NCTM, 2000). So, what can teacher educators and researchers do to maximise the potential of beginning teachers to learn in the contexts of their schools?

Burton and Povey (1996) found that teachers who were aware of their own philosophical position and evaluated their own teaching in relation to it, were more likely to be able to develop in their teaching. This required reflection both in practice and on practice and enabled teachers to act authentically: that is, to teach according to their own beliefs rather than in accordance with the norms and culture of their schools and the wider professional environment. In this way, reflection works alongside enculturation, enabling the teacher to learn through a community of practice in a *critical* way. Such critical reflection may lead to changes in beliefs, practices, or both, in a way that conserves authenticity between the two.

The rationale for my study is based on the premise that critical reflection and participation in communities of practice, as well as the interrelationship between the two, are both important influences on teacher development. It was my contention therefore that an effective way to develop content knowledge for teaching mathematics would be to support teachers in their reflections on their mathematics teaching in a way that enables them to develop within communities of practice. There was also recognition that, alongside developments in content knowledge, related changes in beliefs and practices will need to reflect one another in order to conserve authenticity.

The approach taken in my study is consistent with the idea that teachers' mathematical content knowledge needs to be considered in the context of teaching practice. It draws on both the *enculturation into the professional culture* model and the *professional development through reflection* model of development. Central to my study is the idea that reflection *in* and *on* practice, with a focus on mathematics content knowledge, will promote change in teachers' content knowledge, as well as their conceptions about mathematics and mathematics teaching. The instrument used to promote such reflections in my study has been the Knowledge Quartet framework (Rowland, Huckstep and Thwaites, 2005). This framework helps teachers to focus their reflections on their mathematical content knowledge as evidenced in practice.

The study

My project is a four-year longitudinal study working with a group of beginning primary school teachers to investigate changes in their mathematics teaching over four years. My intention was to help teachers recognise aspects of their mathematics content knowledge revealed through their practice, and through discussion of that practice to enhance these teachers' understanding and development of their own mathematics teaching. The focus on content knowledge encompassed Shulman's notions of both subject matter knowledge (SMK) and pedagogical content knowledge (PCK). This was an evaluative study, in that I set out to evaluate the efficacy of the Knowledge Quartet as a framework for helping teachers reflect on and develop the mathematical content of their teaching, as well as to evaluate the validity of the Knowledge Quartet itself. It was also investigative, in that I hoped to investigate how beginning teachers change in relation to their mathematics teaching and what factors, along with participation in this project, might be influential in this change.

I began from the position that all involved in the project would have their own interpretations of any observed teaching episodes and of the ways in which mathematics content knowledge is revealed through these episodes. Discussion was seen as a means for both revealing and bringing together these interpretations in order to develop the understanding of both. The Knowledge Quartet framework was used in the study as a tool to focus reflections and discussions on mathematics content knowledge and to provide shared understandings that would facilitate communication.

The relationship between the teacher participants and myself as researcher was considered to be that of co-learners (Wagner, 1997) and elements of an action research approach (Stenhouse, 1975; Kemmis and McTaggart, 1988) were employed. Research suggests that any development of teachers' conceptions and related practices is a long term process, and this was recognized in the longitudinal design of my research project.

Methods

The doctoral thesis itself will draw mainly on four case studies. These case studies involve teachers who were all student teachers in the 2004-5 cohort of the one-year early years (3-7 years) and primary (5-11 years) PGCE course at the University of Cambridge. The study began with 12 participants, reducing, as anticipated, to 9 in the second year, then 6 in the third year, and finally 4 in the fourth and final year of the study. All participants were observed teaching during the final placement of their training year, twice during their first year of teaching and three times during their second year of teaching. These lessons were all videotaped and analysed using the categories and codes of the Knowledge Quartet. In the training year, the videotapes were the basis for stimulated recall discussions using the Knowledge Quartet to focus on the mathematical content of the lesson.

During the first year of teaching, feedback using the Knowledge Quartet was given following the two observed lessons. Participants were then sent a DVD of their lesson on which they were asked to write reflections. In the second year of their teaching, only minimal feedback was given following the lesson, as I wanted to see how the teachers would independently make use of the Knowledge Quartet in their reflections. They were sent DVDs of their three lessons and wrote reflections on each of these lessons under the four headings of the Knowledge Quartet. Participants also wrote, and sent me, regular reflections on their mathematics teaching. Over the four years, the Knowledge Quartet framework was increasingly utilised by participants to organise these written reflections.

The use of reflection within the study does not ignore socio-cultural models of teacher development. Individual reflections of participants were made within the communities of practice in their schools and also within a ‘research group community of practice’ made up of the participant teachers and me. Group meetings of all participants were seen as offering a significant support to development as well as a forum for collecting data on the teachers’ perspectives. These meetings took place at the end of the training year and the first year of teaching, and at the end of each term in the second year of teaching.

This is consistent with Julian Williams’ (2007) suggestion that seeing teacher knowledge as held collectively offers a “possible remedy” for developing this knowledge through working with communities of teachers as a “collective subject” (p. 10). The role of interactions with pupils and colleagues in school, and the role of CPD courses, was recognised within the design of my study. All of these were investigated in the one-to-one post-lesson discussions and interviews, as well as in the group discussions.

This year, the fourth and final year of the project, each of the remaining four teachers will be interviewed individually three times - once each term. There will be group meetings twice during the year. Case studies are being built from observations of teaching, discussions following observed lessons, contributions to group meetings, written reflections and individual interviews. For an overview of data collection throughout the project see Appendix 1 - Table of data collection.

Analysis of data

A number of constructs have emerged from the data, which offer insights into what Robert Stake (2006) termed, the condition being studied, or the *quintain*. In this case the *quintain* is the developing mathematics teaching of the beginning teachers. These constructs have been built on existing theories, as suggested by Yin (2003). Such theories include socio-cultural theories of teacher development as well as theories that envisage development through critical reflection of practice. These constructs or themes emerged from the analysis of data from post-lesson interviews, written reflections, group interviews and individual interviews. Stake (2006) suggests that when carrying out multiple case study analysis, attention must first be paid to the rich data of individual cases before assessing the relevance of each case to the *quintain*. The NVivo qualitative analysis software package has been used to facilitate such analysis, first within, and then between cases. Patterns emerging from individual cases and between cases have given rise to a number of codes or ‘nodes’ using NVivo. These codes have been organised hierarchically into themes or ‘tree nodes’. Unsurprisingly, one overarching theme that has arisen from this analysis is that of *reflection* and another is that of *socio-cultural influences*. I am at an early stage in analysing and making sense of all the data. However, the interrelationship between individual critical reflection and participation in communities of practice seems to be emerging as an important area for further exploration.

Initial findings in relation to the individual-social nature of teacher development

Giving participants the Knowledge Quartet framework to focus and deepen their reflections has positively affected the way in which all participants have engaged with learning situations in their own contexts.

> You don't take your teaching for granted. You think about all the images, prompts or examples you'll need. You think as you are teaching of extra aids, how you are phrasing explanations. I think the Knowledge Quartet has pushed me to think from the other side and see more clearly how the children see and what they need. (From written reflection at the end of Amy's second year in post)

> I think the categories are very useful … they kind of give you a way of thinking about what would be a kind of sensible remark to make about your maths lesson, and they evaluate it against certain things. (From group meeting at the end of the first term of Kate's second year of teaching)

Initial findings however, show a contrast in the degree to which teachers have opportunities to participate in communities of practice. These findings are presented in more detail in a paper proposed for PME32 (Turner, 2008). One teacher, Amy, has made a number of comments suggesting that her development is influenced by her colleagues.

> I have found talking the feedback (from me) over with my colleagues, means we have had more of a dialogue about maths and our teaching of maths in school, well in the lower school, with my colleagues and that seems really interesting and useful and it is always good to talk about other people's … about how you are teaching or about how you can move forwards. (From group meeting at the end of Amy's first year in post)

> And a lot of it is from talking to colleagues as well. You do learn from discussions, you're having constant discussions and you learn that way as well. (From the interview with Amy at the beginning of her third year of teaching)

Kate, on the other hand seems to be finding less support from her colleagues.

> I am a bit confused about empty number lines at the moment. When you look at 'The Power of the Number Line' DVD and various numeracy strategy materials, people seem to use them in a 'come and show me how you are going to use this in your own way' kind of approach. However my colleague who previously taught year 3/4 at our school believes that we should only be teaching 'counting on' along the empty number line for subtraction because that is what the children will be taught in year three. (From Kate's written reflections in the middle of her second year in post)

> Occasionally, not very often but occasionally, I see with the other people I am planning with, the other teachers that they don't seem to understand something that I think is quite necessary to understand, and they have either got it wrong or they don't seem to realise that it is important. (From group meeting at the beginning of her second year in post)

Jaworski (2001) and Lerman (2001) suggested that reflection is most effective when shared with others. However, there is evidence from the study that reflective individuals, such as Kate, are able to learn and develop independently of the communities of practice they find themselves in.

> Not very often (have deep conversations about the use of representations), no, not as often as we should because nobody wants to do the planning again. Um, I guess I would just use the other representation rather than discussing it with anybody. (From interview with Kate at the beginning of her third year of teaching)

> As mentioned under foundation, I changed the plans for one day as I didn't agree with the idea behind our investigation. I haven't discussed this with my colleagues as I didn't want to be awkward. (From written reflections at the beginning of Kate's second year of teaching)

Kate is not being simply 'enculturated' into the professional culture she finds herself in. Where there is dissonance between her own beliefs about teaching mathematics and the practices of the community, of which she is a part, she does what she believes to be best. Barbara Jaworski (2006) offers a way of interpreting Kate's behaviour within Wenger's (1998) conceptualisation of *identity* within a *community* *of practice*. One characteristic of identity, as belonging to a community of practice, is that of *alignment* with that community. Jaworski modified Wenger's notion of alignment to that of *critical alignment*, allowing for critical evaluation of the practices of the community while aligning oneself with it. Though not sufficiently confident to explicitly question the ideas and practice of others, the beginning teachers continue to do so in their reflections and in relation to their own practice. They may develop feelings of belonging to the community of practice while continuing to be critical of some of the practices of the community. Their alignment might therefore be described as *critical alignment*, the critical nature of which depends on an ability to reflect independently in a critical way.

It is difficult for teacher educators to influence the learning contexts in the schools of beginning teachers. It is, however, possible to help teachers become reflective practitioners who are able to engage more effectively with these situations. The ability to be critically reflective would seem to be especially beneficial where communities of practice do not support learning in what might be considered, by bodies such as NCTM, to be in a positive direction.

REFERENCES

Alexander R., Rose J., and Woodhead C. (1992) *Curriculum Organisation and Classroom Practice in Primary Schools*. London: Department for Education and Science.

Ball, D. L. and Bass, H. (2003) Toward a practice-based theory of mathematical knowledge for teaching. In B. Davis and E. Simmt (Eds.), *Proceedings of the 2002 annual meeting of the Canadian Mathematics Education Study Group* (pp. 3-14). Edmonton, Alberta, Canada: Canadian Mathematics Education Study Group

Ball, D. L., Hill, H. C. and Bass, H. (2005) 'Who knows mathematics well enough to teach third grade, and how can we decide? *American Educator. http:www.aft.org/pubsreports/American_educator/issues/fall2005/BallF05.pdf*

Ball, D. L., Thames, M. H. and Phelps, G. (submitted) Content knowledge for teaching: What makes it special?

Brown, T., McNamara, O. Jones, L. and Hanley, U. (1999) Primary Student Teachers' Understanding of Mathematics and its Teaching. *British Education Research Journal* 25 (3) pp. 299-322.

Burton, L. and Povey, H. (1996) Competency in the Mathematics Classroom: The Example of Equal Opportunities. In David Hustler and Donald McIntyre *Developing Competent Teachers: Approaches to Professional Competence in Teacher Education.* (pp. 128-140) London, David Fulton.

Calderhead, J. and Shorrock, S. B. (1997) *Understanding teacher education: Case studies in the professional development of beginning teachers.* London: Falmer.

Cobb, P. and McClain, K. (2001) An approach for supporting teachers' learning in social context. In Fou-Lai Lin and Thomas Cooney (Eds) *Making sense of mathematics teacher education.* (pp. 207-231) Dordrecht: Kluwer.

Cobb, P. Yackel, E. and Wood, T. (1991) Curriculum and teacher development: Psychological and anthropological perspectives. In E. Fennema, T. P. Carpenter and S. J. Lamon (Eds), *Integrating research on teaching and learning mathematics.* (pp. 83- 120) Albany, NY: SUNY Press.

Cockcroft, W. H. (1982) *Mathematics Counts: Report of the committee of inquiry into the teaching of mathematics in schools.* London: Her Majesty's Stationary Office.

DfEE (1999) *The National Numeracy Strategy: framework for teaching mathematics from Reception to Year 6*. Sudbury: DfEE Publications.

Fennema, E. and Franke, M. (1992) Teacher knowledge and its impact. In Grouws, D. (Ed) *A Handbook of Research on Mathematics Teaching and Learning.* (pp. 147-164) National Council of Teachers of Mathematics.

Glaser, B. G. and Strauss, A. L. (1967) *The Discovery of Grounded Theory: Strategies for Qualitative Research*. New York: Aldine de Gruyter.

Greeno, J.G. (1998) The situativity of knowing, learning and research. *American Psychologist* (January), pp. 5-26.

Haylock, D. (2006) *Mathematics Explained for Primary Teachers*. 3rd edn. London: Paul Chapman.

Hegarty, S. (2000) Teaching as a knowledge-based activity, *Oxford Review of Education* 26(3-4) pp. 451-465.

Jaworski, B. (1998) Mathematics teacher research: process, practice and the development of teaching. *Journal of Mathematics Teacher Education*, 1 pp.3-31.

Jaworski, B. (2001) Developing mathematics teaching: Teachers, teacher educators and researchers as co-learners. In F-L Lin and T. Cooney (Eds) *Making sense of mathematic teacher education.* (pp. 295-318) Dordrecht: Kluwer.

Jaworski, B. (2007) Theory and Practice in Mathematics Teaching Development: Critical Inquiry as a Mode of Learning in Teaching. *Journal of Mathematics Teacher Education* 9(2) pp. 187-211.

Kemmis, S. and McTaggart, R. (1988) (Eds) The *Action Research Planner* (second edition) Geelong, Victoria: Deakin University Press.

Kennedy, M. (1990) A survey of recent literature on teachers' subject matter knowledge. Issue Paper IP 90-3. Michigan State University: National Center for Research on Teacher Learning

Lave, J. (1988) *Cognition in Practice*. Cambridge: Cambridge University Press.

Lave, J. and Wenger, E. (1991) *Situated learning: Legitimate peripheral participation*. New York: Cambridge University Press.

Lerman, S. (2001) A review of research perspectives on mathematics teacher education. In Fou-Lai Lin and Thomas Cooney (Eds) *Making sense of mathematic teacher education*. (pp. 33-52) Dordrecht: Kluwer.

Ma, L. (1999) *Knowing and teaching mathematics: teachers understanding of fundamental mathematics in China and the United States*. Mahwah, NJ: Lawrence Eribaum.

National Council of Teachers of Mathematics (2000) *Principles and standards for school mathematics.* Reston, VA: Author.

Ofsted (2003) *The National Literacy and Numeracy Strategies and The Primary Curriculum*. London: Office for Standards in Education.

Petrou, M. (2007) ‘Michigan research on developing a practice-based theory of content knowledge of teaching.’ http://www.maths-ed.org.uk/mkit/Petrou_Nuffield_MKiT_270907.pdf

Putman, R. and Borko, H. (2000) What do new views of knowledge and thinking have to say about research on teacher learning? *Educational Researcher*, 29(1), 4-15.

Rowland, T. (2007) Developing knowledge for mathematics teaching: a theoretical loop. In S. Close, D. Corcoran and T. Dooley (Eds.) Proceedings of the second national conference on research in mathematics education (pp. 13-26). Dublin: St Patrick’s College.

Rowland, T. (in press) Researching teachers’ mathematics discipline knowledge. In P. Sullivan (Ed.) *Handbook of Mathematics Teacher Education* Volume 1. Rotterdam: Sense Publishers.

Rowland, T., Huckstep, P., and Thwaites, A. (2005) Elementary teachers’ mathematics subject knowledge: the knowledge quartet and the case of Naomi. *Journal of Mathematics Teacher Education,* 8(3), 255-281

Schön, D.A. (1983) *The Reflective Practitioner*, New York, Basic Books.

Shulman, L (1986) Those who understand, knowledge growth in teaching. *Educational Researcher* 15 (2), pp. 4-14.

Shulman, L. (1986) Those who understand: knowledge growth in teaching. *Educational Researcher* 15(2), 4-14.

Stake, R. E. (2006) *Multiple case study Analysis*, New York: Guilford Press.

Stenhouse, L. (1975) *An introduction to curriculum research and development*. London: Heinemann.

Strong, M. and Baron, W. (2004) An analysis of mentoring conversations with beginning teachers: suggestions and responses. *Teaching and Teacher Education*, 20, 47-57

Turner, F. (2008) *Growth in teacher knowledge: individual reflection and community participation.* Paper proposed for the joint meeting of the International group and the North American chapter of Psychology of Mathematics Education (PME32) Morelia, Mexico July 2008.

Wagner, J. (1997) The unavoidable intervention of educational research: A framework for reconsidering researcher-practitioner cooperation. *Educational Researcher* 16 (7) pp. 13-22.

Wenger, E. (1998) *Communities of practice: Learning meaning and identity*. Cambridge: Cambridge University Press.

Williams, J. (2007) *Audit and evaluation of pedagogy: towards a sociocultural perspective.* Paper for the 'Mathematical Knowledge in Teaching', Nuffield Foundation seminar series. Seminar 3, London.

Yin, R. K. (2003) *Case study research. Design and methods*. Third edition. London: Sage.

Appendix 1

Table of data collection

Year	**Y1** **Training year**	**Y2** **NQT**	**Y3** **ECT**	**Y4** **ECT**
Date	**May – July 2005**	**Sept 2005** **– July 2006**	**Sept 2006** **– July 2007**	**Sept 2007** **– July 2008**
Number of participants	11	9	6	4
Research activity – data collection (per participant)	Video-tape 1 mathematics lesson Analysis by KQ Video-stimulated recall and feedback End of year group interview	Video-tape 2 mathematics lessons Analysis by KQ Feedback End of year group interview	Video-tape 3 mathematics lessons (one per term) Analysis by KQ 3 end of term group interviews	3 interviews (one per term) 2 group interviews
Participant data provision	Some lesson reflections	Sporadic reflections on individual lessons and mathematics teaching generally	Reflections on observed lessons Half-termly reflections on mathematics teaching	Half-termly reflections on mathematics teaching