Mathematical knowledge in teaching: the case of argumentation and proof

Discussion Group theme – auditing and assessing mathematical knowledge in teaching

We grappled with what assessing proof, or proving, would look like, in ITE for example. An example was given from a LOGO activity in which students (trainees) are asked to draw an equilateral triangle (or some other regular polygon, apart from a square). The error of turning through 60^o rather than 120^o is commonplace. But what does this show, or indicate? Maybe something about knowledge of sums of interior or exterior angles of polygons? Very probably not having perceived that the turtle turns through the latter. But knowledge of proof? Another suggestion was about *n*th terms of sequences, related to configurations of some sort. The thought here seemed to be about 'structural generalisation' as opposed to pattern spotting. But knowledge of proof?

The existence of proof-modes other than 'formal' proof was raised, and briefly discussed. What about 'proofs without words'? Maybe having a repertoire of such proofs is (at least) as important as competence with the formal, symbolic-general mode. Someone mentioned the pedagogical role of generic examples. Another person observed that it is a question of *meaning*, and the goal(s) of proof. Formal, less formal, informal – what are they *for*. This led to an extended discussion of proof(s) that the sum of two odd numbers is even. A situation, from an ITE session, was described, in which a student wrote of the board:

1+2=3
2+3=5
3+4=7
etc (several lines)

These were examples, instances of the 'result', organised in a way intended to offer explanatory insight. [TR: It occurred to me later that this could helpfully be embedded in the context of an enquiry into sums of 2 consecutive integers, 3 consecutive integers, 4 consecutive integers …]

The point here is that *instances* of the result (*odd+odd=even*) are purposefully structured so as to hint at meaning beyond the individual statements of fact.

Another person recalled teaching trigonometric identities to a sixth form class, offering not just results, but also proofs. Judging by the students' surprised reaction, they viewed her as "an alien, from another planet".

We concluded with a brief reference to our group 2 'brief', when someone offered their sense that 'audit and assess' had a static feel to it, in contrast to the dynamic notion of 'developing knowledge in teaching'. A suitable cue for the next seminar?

Tim Rowland, January 2008

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