The Situated Nature of Mathematics Teacher Knowledge

Jeremy Hodgen
King's College London

In this paper, I examine the issues of mathematics teachers' subject knowledge from the perspective of what Lerman (2000) terms the "social turn" in mathematics education. A key work in this social turn is Lave and Wenger's (1991) monograph examining the nature of learning as apprenticeship and re-casting knowledge in terms of situated cognition[1]. Whilst this work largely considered learning in informal settings outside formal education, it has nevertheless been influential in mathematics education (Boaler, 2002; Greeno, 1998), particularly in relation to the perennial issue of how students use, or transfer, the mathematics learnt in school into real world contexts. This notion of transfer has in turn been the subject of much contentious debate (e.g., Anderson, Reder, & Simon, 1996; Greeno, 1997), although it is arguable that this debate has often been characterised more by misunderstandings than by genuine disagreement (Anderson, Greeno, Reder, & Simon, 2000). As Putnam and Borko (2000) argue,

> It is easy to misinterpret scholars in the situative camp as arguing that transfer is impossible—that meaningful learning takes place only in the very contexts in which the new ideas will be used. The situative perspective is not an argument against transfer, however, but an attempt to recast the relationship between what people know and the settings in which they know—between the knower and the known. (p.12)

From this perspective, knowledge is social and contextualised rather than individual and general, whilst knowledge about mathematics teaching is less about general principles and more about "intertwined collections of more specific patterns that hold across a variety of situations" (Putnam & Borko, 2000, p.13). Hence, it is a recognition of the similarities and differences between these patterns that enables transfer of knowledge between settings.

But what does this mean for our conceptualisations of mathematics teacher knowledge? In order to address this question, I will first the theoretical literature on mathematics teacher knowledge more generally and how the situated perspective relates to this literature. Here, I follow Putnam and Borko (2000) in regarding the situated perspective not as a fundamental shift in thinking about education but rather as continuing an interest in processes that has its roots in the ideas of educational theorists such as Dewey, Vygotsky and others.

---

[1] Recently, there has been a growing interest in other "social" theories such as Activity Theory (Engeström, Miettinen, & Punamaki, 1999) and Discourse Analysis (e.g., Gee, 1999). Both Gee and Wenger (1998) see considerable similarities in these approaches in particular in their conceptualisation of knowledge.

THE PROBLEM OF MATHEMATICS TEACHER KNOWLEDGE

It appears self-evident that teachers should know about mathematics in order to teach it effectively. But, teacher knowledge in mathematics is an area of some controversy. There is evidence that poor subject knowledge in mathematics has a negative impact on teaching (e.g., Bennett & Turner-Bisset, 1993; McDiarmid, Ball, & Anderson, 1989; Rowland, Martyn, Barber, & Heal, 2000). There is considerably less consensus on what constitutes the mathematical knowledge necessary for teaching. Some have argued that improving teachers' knowledge of mathematics per se will lead to better teaching (e.g., Alexander, Rose, & Woodhead, 1992). However, the evidence base in this area suggests otherwise. Several studies, for example, have found no link between teachers' mathematical knowledge as measured in terms of academic mathematical qualifications and effective teaching (Askew, Brown, Rhodes, Johnson, & Wiliam, 1997; Begle, 1968, 1979; Eisenberg, 1977). What is clear is that the connection between teacher knowledge and teaching outcomes is neither simple nor straightforward and that further research in this area is needed.

To deal with this problem, research has focused on exploring the nature of teacher knowledge in mathematics. One strand of this research has been to link mathematical knowledge for teaching to ways of knowing in the discipline of mathematics. Lampert (1986), for example, distinguishes between *procedural* and *principled* knowledge of mathematics. Procedural knowledge is a rule guided "knowing that" and concerns mathematical procedures and their use to compute correct answers. Principled knowledge on the other hand is a wider and more conceptual "knowing how" and includes the knowledge of mathematical concepts that enable the construction of procedures for solving mathematical problems. Lampert's distinction has similarities with has similarities to Skemp's (1976) distinction between instrumental and relational understandings, Prestage and Perks' (2001) learner-knowledge and teacher-knowledge, and Thompson, Philipp, Thompson, & Boyd's calculational and conceptual orientations (1994).

Increasingly, however, researchers have been arguing that mathematical knowledge for teaching is distinct and different to the knowledge necessary to practice mathematics. Much of this work builds on Shulman's (1986) notion of *pedagogical content knowledge* which "goes beyond the subject per se to the dimension of subject knowledge *for teaching* … the particular form of content knowledge that embodies the aspects of content most germane to its teachability" (p. 9, original emphasis). In her study of elementary mathematics teachers in China and the US, Ma (1999) provides a useful analysis clarifying the principled nature of pedagogical content knowledge in primary mathematics, which she refers to as *profound understanding of fundamental mathematics*. Similarly, Ball and Bass (2000) point to the highly abbreviated character of mathematics and see pedagogical content knowledge in terms of "unpacking" mathematics.

The nature of pedagogical content knowledge is itself, however, something of a contested idea within the education research community. McNamara (1991), for example, argues that there is no clear distinction between subject knowledge and pedagogical content knowledge. Indeed, Corbin and Campbell (2001) argue that pedagogical content knowledge is most useful as a metaphor that locates teacher knowledge embedded within the complex and unpredictable practice of teaching. Another critique is epitomised by Brown and McIntyre (1991), who argue that much of a teachers' knowledge is tacit, craft knowledge that cannot be codified as theoretical abstract knowledge. For Brown and McIntyre, the knowledge of an expert teacher is more intuitive and in a very real sense less explicit than that of a novice.

Taking this notion of tacit knowledge further, situated theorists problematise the very nature of knowledge arguing that teacher's mathematical knowledge, like any other form of knowledge, is located in social practice (Greeno, 1998; Putnam & Borko, 2000). Hence, Adler (1998) refers to a dynamic and contextualized process of knowing "knowing" rather than the more static and abstract notion of "knowledge", whilst Putnam and Borko (adapted from 2000) argue that teachers' knowledge is situated, social and distributed. To illustrate what this actually means for mathematics teacher knowledge, I use a case study drawn from my own research.

A PRIMARY TEACHER'S MATHEMATICAL KNOWLEDGE: SITUATED, SOCIAL AND DISTRIBUTED

In this section, I discuss the case of Alexandra[2], a primary teacher, and her knowledge of proportional reasoning. I contrast Alexandra's knowledge of proportional reasoning in the context of developing lessons and leading professional development sessions with her knowledge in the context of a structured mathematics interview.

This case study is drawn from on a four year longitudinal study into the professional change of six teachers involved as teacher-researchers in the Primary Cognitive Acceleration in Mathematics Education (CAME) Project research team (Johnson, Hodgen, & Adhami, 2004). The Primary CAME project research team consisted of four researchers, four teacher-researchers and the Local Education Authority mathematics advisor. During the school year 1997/8, the research team met fortnightly to develop Thinking Maths lessons specifically for primary children aged 9 – 11 (Years 5 and 6). During a second of the project, over the school years 1998/9 and 1999/2000, a further cohort of teachers from seven more schools joined the project to begin implementing the Thinking Maths lessons more widely. In Phase 2, the teacher-researchers led professional development sessions for the new cohort of teachers aimed at enabling this cohort to teach the Thinking Maths lessons in their own classes.

In the research study, data collection was qualitative using multiple methods, including observations of seminars, lessons and PD sessions, and interviews with individuals and groups, and structured mathematical interviews (adapted from Millett, Askew, & Simon, 2004). (See Hodgen, 2003 for more detail on the study.) During this interview teachers were asked to solve several problems and to suggest model, stories or diagrams to use when teaching the ideas to children. The questions themselves largely related to aspects

[2] Alexandra is a pseudonym.

of mathematics within the KS1 and 2 Framework for teaching mathematics (DfEE, 1999). Here, I focus on three related questions:

$$0.5 \times 0.2, \qquad 3 \div 0.75, \qquad 1\,{}^{3}/_{4} \div {}^{1}/_{2}.$$

Here, I was particularly interested in the extent to which the teachers could generate a variety of appropriate and pedagogically useful illustrations, and in the range of different meanings of multiplication and division that they drew upon. Ma (1999), for example, describes three models of division: measurement; partitive; and, factors and product. These broadly relate the understandings of multiplication in terms of repeated addition, scaling and arrays (or areas) in the Mathematics Framework (DfEE, 1999). There is extensive research evidence to suggest that the area model is use in only limited ways in UK mathematics classrooms (Nunes, 2001).

As a Primary CAME teacher-researcher, Alexandra was involved in the development of a number of lessons addressing students' misconceptions in collaboration with other teacher-researchers and academics. Specifically, together with another teacher, she developed two lessons focusing on fractions: "Share an Apple", and "Halving & Thirding" (Johnson et al., 2003). In "Share an Apple", the focus is on representations and comparisons of fractions. So, for example, children are asked to consider various ways of representing and comparing the magnitude of simple fractions of everyday objects. In Halving & Thirding, the focus is on developing and connecting different representations for the multiplication of fractions, including repeated multiplication by ${}^{1}/_{2}$ and ${}^{1}/_{3}$, with a particular focus on developing the area model for multiplication. Alexandra herself suggested this focus on the area model based on her experiences of team teaching the lesson. She also led the PD sessions introducing these lessons and had contributed to an academic paper on the development of these lessons. (See Hodgen & Johnson, 2004, for a discussion of Alexandra's development as a teacher of mathematics during the project.)

For much of the period of her participation in Primary CAME, Alexandra was also a Numeracy Consultant, whose responsibilities included delivering training aimed at enhancing primary teachers subject knowledge of mathematics. In this role, I observed her teach several National Numeracy Training training sessions on both fractions and multiplication. In addition, in response to her perceptions of weaknesses in these training materials and in collaboration with another Numeracy Consultant, she developed a further session for teachers in which she focused on the use of the area model of multiplication in relation to fractions together with the concept of equivalence (Hodgen, 2003).

Given these experiences, I had expected Alexandra to demonstrate a sophisticated understanding of multiplication in the mathematics interview. Yet, her knowledge appeared to be very significantly weaker in this setting: she appeared to know "less" and to know it less securely.

Alexandra could successfully answer all the questions performing most of the necessary calculational procedures correctly, although on several questions this took a considerable amount of time and whilst solving the problems she made several mistakes which she corrected during the interview. At one point she indicated some awareness of her limited understanding referring to division by fractions [$1^{3}/_{4} \div {}^{1}/_{2}$] as follows: "If I was doing that the way I was taught to do it, I would just turn that all upside down. And I have real problems with this idea of division by fractions." However, she was unable to carry out this procedure and solved the question by converting to decimals mentally then using a calculator. To solve 0.5 x 0.2, she used a standard multiplication algorithm, as in Figure 1, commenting on how she knew where to place the decimal point in the product: "There are two decimal places in the question, so there must be two decimal places in the answer." This, together with her inclusion of the multiplication by zero, strongly suggests that her understanding of this method is certainly heavily reliant on procedural knowledge.

		0	.	5
x		0	.	2
		1		0
0		0		0
0	.	1		0

Figure 1: Alexandra's procedure for solving 0.5 x 0.2

Although Alexandra read the answer correctly as 0.1 and used the same form as in the question, she did not notice that this could be read as a tenth or that the calculation was equivalent to either of the relatively simple "half of two tenths" or "half of a fifth." Hence, she appeared to have no strategy to check or make sense the result of this calculation procedure. Indeed, she could not generate an illustration of this problem. Whilst she did not get this problem "wrong", her knowledge did appear to be partial and limited.

Alexandra found the generation of any models extremely difficult and required considerable support and prompting to tackle these questions. Indeed, she asked me, with apparent disbelief, if I could do it. She provided a single story for just two of the three problems. Reflecting her preference for decimal fractions, she found 3 ÷ 0.75 relatively straightforward after I had suggested thinking about contexts involving measures: "how many lots of seventy five pence can you get from three pounds." However, she had considerable difficulty with $1^{3}/_{4} \div {}^{1}/_{2}$, eventually producing the following story:

> If you said that was one, and that was three quarters you'd get three halves and half a half out of it. But that's not very helpful is it? … One, OK, that's one and three quarters, so you can get one, two, three. Three halves out of it. And half of a half.

Whilst this story certainly provides an illustration of $1^{3}/_{4} \div {}^{1}/_{2} = 3^{1}/_{2}$, it is a re-statement of the problem in terms of repeated addition / subtraction[3]. Alexandra had developed the

[3] It is interesting that this model reflects the only occurrence of division by fractions in the Mathematics Framework: "How many halves in $3\,{}^{1}/_{2}$?" (DfEE, 1999, Y456 examples, p. 25).

two fractions lessons with the specific aim of enabling children to develop a range of models for the representation of fractions. The Halving & Thirding lesson had used both measurement and area representations for the multiplication of fractions, an aspect of the lesson which she herself had highlighted several times during the lesson simulation to Phase 2 teachers. It is somewhat surprising that, given these fairly intense lesson development experiences, together with her experiences as a Numeracy Consultant, she could not transfer the area model to division by fractions or, more significantly, to the multiplication of decimal fractions. Indeed, she was unable to provide an illustration of 0.5 x 0.2. More surprising still is her reaction to being asked to think of models, given that I had observed her emphasise different meanings of multiplication and division, including repeated subtraction / addition and the area / array models, and the need to understand children's different ways of seeing mathematical relationships when leading training sessions.

Of course, this does not mean that she did not know other models. However, the difficulty that she encountered generating these stories does suggest that she lacked an intuitive familiarity with these and different models of multiplication / division. Alexandra's failure to draw on her experiences of developing the fractions lessons suggests that her knowledge was *situated*. Faced with these interview problems in other situations, Alexandra "knew" more and performed "better" but relatively "strong". It is important to recognise that the testing nature of the interview did exacerbated her difficulties and that Alexandra perceived the interview as something of a threat to her professional, although the roles of teacher-researcher and Numeracy Consultant both involved situations in which Alexandra's subject knowledge was under both implicit and explicit scrutiny.

Alexandra's mathematical knowledge was also both *social* and *distributed*. She "knew", for example, about different models for the multiplication of fractions in the context of lesson development and, as a tutor during INSET sessions, when such knowledge was explicitly part of her role. Significantly, these were settings where she was working in collaboration with others and she had access to lesson or course guidance. She was not simply a passive participant in these contexts nor was she simply "delivering" the course materials. In fact, in both settings, Alexandra's knowledge appeared to be strong – in the sense that she could participate in the discussions within the research team and respond to teachers' questions. Alexandra's knowledge was social in the sense that these discourse communities provided the cognitive tools with which to be knowledgable. It was distributed in the sense that it was "stretched over" (Lave, 1988) other individuals and artefacts.

THE CONTRIBUTION OF SITUATED THEORIES

Surprisingly, given the interest more generally in mathematics education, there has been little attention from this perspective on the issues of mathematics teacher knowledge. Moreover, as I have already noted, I consider there to be continuities and connections between the situated perspective and more traditional perspectives on mathematics education. Hence, in considering the contribution of situated cognition, some of the studies that I refer to draw on different but related theoretical perspectives.

There is a widespread recognition about the complexity of mathematics teacher knowledge as discussed above. Whilst the situated perspective certainly helps to understand particular facets of this complexity, its particular contribution relates to the conceptualisation of knowledge and its application. Viewing knowledge as situated, social and distributed places greater emphasis on the communities in which mathematics teachers are engaged rather than on individual knowledge. Building on Spillane's (1999) work, Millett, Brown and Askew (2004) highlight the importance of the professional community of teachers in a school. In the Leverhulme Numeracy Research Programme, some primary schools appeared to successfully "share" mathematics knowledge and expertise amongst a groups of teachers through a mathematics co-ordination team (Millett & Johnson, 2004).

I referred above to the notion of transfer, suggesting that this might be conceived of in terms of the recognition of similarities and differences between settings. Lave (1992), for example, argues that much problem-solving in schools is not authentic – in contrast to the messy and complex problems of the real world, school mathematics problems tend to be straightforward and routine. But, Putnam and Borko (2000) argue that the problem of authenticity is related to the authenticity of learning rather than necessarily to the authenticity of problems themselves: "Authentic activities foster the kinds of thinking and problem-solving skills that are important in out-of-school settings, whether or not the activities themselves mirror what practitioners do" (p.4-5). This highlights the two-fold problem of authenticity in mathematics. Mathematics teaching involves to stages of recontextualisation of mathematics knowledge – a recontextualisation of teachers own mathematics learner knowledge for the classroom to enable students recontextualise this classroom mathematics for out-of-school contexts.

Lampert (1998) argues that the communities of teachers and academics can be seen as distinct communities with different forms of discourse and different ways of warranting knowledge. Hence, teacher educator and academic knowledge about mathematics education, like mathematics teacher knowledge, are both social and situated and the process of teacher education involves the recontextualisation of this knowledge. (See Begg, Davis, & Bramald, 2003, on the difficulties involved in this process.)

A second contribution relates to the nature of learning. Adler (1998) argues that becoming a mathematics teacher involves learning to talk both *within* and *about* mathematics teaching and learning rather than simply learning new knowledge. In their study a group of mathematics teachers from one middle school, Stein, Silver and Smith (1998) similarly highlight the importance of story and narrative in restructuring and reworking knowledge about mathematics teaching. They see this restructuring of existing knowledge and experience as more important than the acquisition of new knowledge – echoing Askew, Brown, Rhodes, Johnson, & Wiliam's (1997) findings about the importance of teachers' beliefs about mathematics in the teaching of numeracy.

Stein, Silver and Smith (1998) place these notions of story and narrative in the context of teachers' professional identities, arguing that teacher learning is best conceived of as a process of identity change. This focus on identity highlights part of the difficulty of teacher learning. Bartholomew (2006), for example, draws on Hollway and Jeffrey's (2000) notion of the "defended self" to highlight how mathematics teachers may resist learning because they perceive it as a threat to their being. Hodgen and Johnson (2004) discuss teacher motivation and the reasons why teachers participate (or do not participate) in learning about mathematics education, arguing that the motivation to change is inextricably linked to teachers' identities and the social context in which they are located. Elsewhere, Hodgen (2005) highlights the role of desire and imagination in developing and transforming teachers' relationships with mathematics.

In an analysis of students' mathematical identities, Boaler and Greeno (2000) relate Holland, Lachicotte, Skinner, and Cain's (1998) conception of identity to Belenky, Clinchy, Goldberger, & Tarule's (1986) notions of authority and knowing. They link procedural knowing to an acceptance of external authority in mathematics and conceptual, or principled, knowing to a more questioning and critical stance - the need to "know why". Povey, Burton, Angier, & Boylan (1999) demonstrate how developing an authorial stance towards mathematics enables teachers to develop such a critical stance. In a discussion of Alexandra's professional change, the teacher considered above, Hodgen and Johnson (2004) argues that providing Alexandra with space and opportunity to engage in reflection and, thus, develop a more critical relationship with mathematics.

A third contribution relates to the analysis of learning settings. The situative perspective is often seen as providing a critique of current practices in schooling rather than offering an alternative vision (Lerman, 2000)[4]. Greeno's (1998) work, however, provides a useful method of analysing learning situations. He highlights the importance of understanding the constraints and affordances: constraints that enable participants (teachers and learners) to predict and anticipate activities and outcomes; affordances that provide opportunities for participants to draw on practices from elsewhere. Boaler (2000) highlights the importance of the social context of learning. In a reanslysis of her study of open-ended and traditional approaches to school mathematics (2002), she describes how the students, who experienced the open-ended approach, more easily related school mathematics to out-of-school contexts in part because of the similarities in the way mathematics was practiced.

IMPLICATIONS FOR THE PRACTICES OF TEACHING, TEACHER EDUCATION AND DEVELOPMENT

In considering the implications of this situated perspective on mathematics teaching and teacher education I focus on three areas: research on teacher knowledge, the role of relationships and emotion, and issues of collaboration.

[4] See, for example, Lave and Wenger's (1991) rather brief and simplistic critique of school education.

Researching mathematics teacher knowledge

There is an increasing interest in the measurement of teachers' mathematics knowledge and the relationship with student learning (Hill, Rowan, & Ball, 2005; Tatto et al., 2006). That mathematics teachers' subject knowledge is difficult to pin down and codify is evident both from the analysis above and the other papers for this seminar. Almost inevitably, the focus on knowledge is concentrated on the more easily describable ideas (e.g. number facts) with much less emphasis placed on the more ephemeral but equally important ideas such as symmetry (Teacher Training Agency, 2003). However, the situated perspective suggests that problem goes beyond this issue of codification in that teachers' knowledge is not only situated but social and distributed. The testing of individual teachers is likely to focus on decontextualised mathematics knowledge, which, as in the case of Alexandra above, may be very different from their classroom knowledge. Nevertheless, the issue of how mathematics teacher knowledge is enacted and the relationship with classroom practice remains poorly understood and research in this area, like the research in mathematics teacher education generally (Adler, Ball, Krainer, Lin, & Novotna, 2005), is largely limited to small scale studies. Given the analysis above, approaches that focus on the notion of recontextualisation (Adler & Davis, 2006) may offer insights in this area.

Relationships, care and emotion

The problem of maths anxiety amongst teachers, particularly primary teachers, is well documented (Bibby, 1999; Buxton, 1981). However, simply reducing anxiety and enabling teachers to "feel better" about mathematics can lead to complacency (Askew, 1996). Drawing on Noddings' (Noddings, 1992) work on care, Askew and I have argued that teachers knowledge of mathematics is both intellectual and emotional (Hodgen & Askew, 2006). The motivation to do mathematics – or to teach mathematics – is both individual and social. Professional development in primary mathematics has generally focused on cognitive and pedagogic issues: teachers' mathematics subject knowledge, how children learn and teaching approaches (Advisory Committee on Mathematics Education, 2006). These issues are, of course, important. But, the importance of identity in coming to know suggested by the situated perspective suggests that such an approach is doomed to failure unless placed within an affective frame in which teachers have space to question mathematics and mathematics teaching.

Collaboration between teachers and between teachers and researchers

The efficacy of collaborative approaches to mathematics teacher education is well-established (e.g., Clarke, 1994) and the situated perspective lends further theoretical weight to such approaches. Spillane (1999) argues that for professional change of any significance, mathematics teachers need social spaces in which they have access to "rich deliberations about the substance … a practising of reform ideas with other teachers and reform experts includ[ing] material resources or artefacts that support [these] deliberations" (p.171). Collaborative approaches between teachers and academics have been shown to be effective (e.g., Black & Wiliam, 2003; Jaworski, 2003)[5] but such approaches are relatively expensive and ways in which such work can be scaled up is poorly understood.

REFERENCES

Adler, J. (1998). Lights and limits: Recontextualising Lave and Wenger to theorise knowledge of teaching and of learning school mathematics. In A. Watson (Ed.), *Situated cognition and the learning of mathematics* (pp. 161-177). Oxford: Centre for Mathematics Education Research. University of Oxford, Department of Educational Studies.

Adler, J., Ball, D. L., Krainer, K., Lin, F.-L., & Novotna, J. (2005). Reflections on an emerging field: Researching mathematics teacher education. *Educational Studies in Mathematics, 61*(3), 359-381.

Adler, J., & Davis, Z. (2006). Opening another black box: Researching mathematics for teaching in mathematics teacher education. *Journal for Research in Mathematics Education, 37*(4), 270-296.

Advisory Committee on Mathematics Education. (2006). *Ensuring effective Continuing Professional Development for teachers of mathematics in primary schools*. London: ACME/The Royal Society.

Alexander, R., Rose, J., & Woodhead, C. (1992). *Curriculum organisation and classroom practice in primary schools: A discussion paper*. London: Her Majesty's Stationary Office.

Anderson, J. R., Greeno, J. G., Reder, L. M., & Simon, H. A. (2000). Perspectives on learning, thinking and activity. *Educational Researcher, 29*(4), 11 - 13.

Anderson, J. R., Reder, L. M., & Simon, H. A. (1996). Situated learning and education. *Educational Researcher, 25*(4), 5-11.

Askew, M. (1996). Using and applying mathematics' in schools: reading the texts. In D. C. Johnson & A. Millett (Eds.), *Implementing the mathematics National Curriculum: policy, politics and practice* (pp. 99-112). London: Paul Chapman Publishing Ltd.

Askew, M., Brown, M., Rhodes, V., Johnson, D. C., & Wiliam, D. (1997). *Effective teachers of numeracy* (Final report). London: King's College.

Ball, D. L., & Bass, H. (2000). Interweaving content and pedagogy in teaching and learning to teach: Knowing and using mathematics. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 83-104). Westport, CT: Ablex Publishing.

Bartholomew, H. (2006). *Emotion and mathematics professional development in a low socio-economic area in New Zealand.* Paper presented at the Mathematics Education Seminar Series, King's College London, 31st January.

Begg, A., Davis, B., & Bramald, R. (2003). Obstacles to the dissemination of mathematics education research. In A. J. Bishop, M. A. Clements, C. Keitel, J.

[5] Interestingly for this situated analysis, Black and Wiliam (2003) argue that a recognition of the importance of teachers' local knowledge is crucial in the dissemination of research.

Kilpatrick & F. K. S. Leung (Eds.), *Second International Handbook of Mathematics Education* (pp. 591-632). Dordrecht: Kluwer.

Begle, E. G. (1968). Curriculum research in mathematics. In H. J. Klausmeier & G. T. O'Hearm (Eds.), *Research and development toward the improvement of education*. Madison, WI: Dembar Educational Research Services.

Begle, E. G. (1979). *Critical variables in mathematics education: Findings from a survey of empirical research*. Washington, DC: Mathematics Association of America and the National Council of Teachers of Mathematics.

Belenky, M. F., Clinchy, B. M., Goldberger, N. R., & Tarule, J. M. (1986). *Women's ways of knowing: The development of self, voice and mind*. New York: Basic Books.

Bennett, N., & Turner-Bisset, R. (1993). Knowledge bases and teaching performance. In N. Bennett & C. Carre (Eds.), *Learning to teach* (pp. 149-164). London: Routledge.

Bibby, T. (1999). Subject knowledge, personal history and professional change. *Teacher Development, 3*(2), 219 - 232.

Black, P. J., & Wiliam, D. (2003). "In praise of educational research": Formative assessment. *British Educational Research Journal, 29*(5), 623-638.

Boaler, J. (2000). Exploring situated insights into research and learning. *Journal for Research in Mathematics Education, 31*(1), 113-119.

Boaler, J. (2002). *Experiencing school mathematics: traditional and reform approaches to teaching and their impact on student learning.* (2nd ed.). Mahwah, NJ: Lawrence Erlbaum Associates.

Boaler, J., & Greeno, J. G. (2000). Identity, agency and knowing in mathematics worlds. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 171-200). Westport, CT: Ablex Publishing.

Brown, S., & McIntyre, D. (1991). *Making sense of teaching*. Buckingham: Open University Press.

Buxton, L. (1981). *Do you panic about maths? Coping with maths anxiety*. London: Heinemann Educational Books.

Clarke, D. M. (1994). Ten key principles from research for the professional development of mathematics teachers. In D. B. Aichele & A. F. Coxford (Eds.), *Professional development for teachers of mathematics: The 1994 Yearbook of the National Council of Teachers of Mathematics* (pp. 37 - 48). Reston, VA: National Council of Teachers of Mathematics.

Corbin, B., & Campbell, A. (2001). *Pedagogical content knowledge: Metaphor in search of teaching?* Paper presented at the British Educational Research Association (BERA) Annual Conference, 13-15 September, University of Leeds.

DfEE. (1999). *The National Numeracy Strategy: Framework for teaching mathematics from Reception to Year 6*. London: Department for Education and Employment.

Eisenberg, T. A. (1977). Begle revisited: Teacher knowledge and student achievement in algebra. *Journal for Research in Mathematics Education, 8*, 216-222.

Engeström, Y., Miettinen, R., & Punamaki, R.-L. (1999). Activity theory and individual and social transformation. In Y. Engeström (Ed.), *Perspectives on activity theory: Learning in doing* (pp. 19-38). Cambridge: Cambridge University Press.

Gee, J. P. (1999). *An introduction to discourse analysis: Theory and method*. London: Routledge.

Greeno, J. G. (1997). On claims that answer the wrong questions. *Educational Researcher, 25*(1), 5-17.

Greeno, J. G. (1998). The situativity of knowing, learning and research. *American Psychologist*(January), 5-26.

Hill, H., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal, 42*(2), 371-406.

Hodgen, J. (2003). *Teacher identity and professional development in primary school mathematics.* Unpublished PhD thesis, King's College, University of London.

Hodgen, J. (2005). *Motivation, desire and teacher change in primary mathematics.* Paper presented at the The Fifteenth International Commission on Mathematical Instruction (ICMI) Study: The Professional Education and Development of Teachers of Mathematics, Brazil 15-21 May 2005.

Hodgen, J., & Askew, M. (2006). Relationships with/in primary mathematics: identity, emotion and professional development. *Proceedings of the British Society for Research into Learning Mathematics, 26*(2), 37-42.

Hodgen, J., & Johnson, D. C. (2004). Teacher reflection, identity and belief change in the context of Primary CAME. In A. Millett, M. Brown & M. Askew (Eds.), *Primary mathematics and the developing professional* (pp. 219-244). Dordrecht: Kluwer.

Holland, D., Lachicotte, W., Skinner, D., & Cain, C. (1998). *Identity and agency in cultural worlds*. Cambridge, MA: Harvard University Press.

Hollway, W., & Jefferson, T. (2000). *Doing qualitative research differently*. London: Sage.

Jaworski, B. (2003). Research Practice into/influencing Mathematics Teaching and Learning Development: Towards a Theoretical framework based on co-learning partnerships. *Educational Studies in Mathematics, 54*(2-3), 249-282.

Johnson, D. C., Adhami, M., Shayer, M., Hodgen, J., Hafeez, R., Dubben, S., et al. (2003). *Primary CAME Thinking Maths: Teachers' guide*. London: BEAM Education.

Johnson, D. C., Hodgen, J., & Adhami, M. (2004). Professional development from a cognitive and social standpoint. In A. Millett, M. Brown & M. Askew (Eds.), *Primary mathematics and the developing professional* (pp. 185-217). Dordrecht: Kluwer.

Lampert, M. (1986). Knowing, doing and teaching mathematics. *Cognition and Instruction, 3*(4), 305-342.

Lampert, M. (1998). Studying teaching as a thinking practice. In J. G. Greeno & S. V. Goldman (Eds.), *Thinking practices in mathematics and science learning* (pp. 53-78). Mahwah, NJ: Lawrence Erlbaum Associates.

Lave, J. (1988). *Cognition in practice: mind, mathematics and culture in everyday life*. Cambridge: Cambridge University Press.

Lave, J. (1992). Word problems: A microcosm of theories of learning. In P. Light & G. Butterworth (Eds.), *Context and cognition: Ways of learning and knowing* (pp. 74-92). Hemel Hempstead: Harvester Wheatsheaf.

Lave, J., & Wenger, E. (1991). *Situated learning: legitimate peripheral participation*. Cambridge: Cambridge University Press.

Lerman, S. (2000). The social turn in mathematics education research. In J. Boaler (Ed.), *Multiple prespectives on mathematics teaching and learning* (pp. 19-44). Westport, CT: Ablex Publishing.

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Mahwah, New Jersey: Lawrence Erlbaum Associates.

McDiarmid, G. W., Ball, D. L., & Anderson, C. W. (1989). Why staying one chapter ahead doesn't really work: Subject-specific pedagogy. In M. C. Reynolds (Ed.), *Knowledge base for the beginning teacher* (pp. 193-206). Oxford: Pergamon.

McNamara, D. (1991). Subject knowledge and its application: problems and possibilities for teacher educators. *Journal of Education for Teaching, 17*(2), 113-128.

Millett, A., Askew, M., & Simon, S. (2004). Responses of teachers to a course of intensive training. In M. Askew, M. Brown & A. Millett (Eds.), *Primary mathematics and the developing professional* (pp. 127-154). Dordrecht: Kluwer.

Millett, A., Brown, M., & Askew, M. (2004). Drawing conclusions. In M. Askew, M. Brown & A. Millett (Eds.), *Primary mathematics and the developing professional* (pp. 245-255). Dordrecht: Kluwer.

Millett, A., & Johnson, D. C. (2004). The role of the school in developing mathematics. In M. Askew, M. Brown & A. Millett (Eds.), *Primary mathematics and the developing professional* (pp. 57-95). Dordrecht: Kluwer.

Noddings, N. (1992). *The challenge to care in schools: An alternative approach to education*. New York: Teachers College Press.

Nunes, T. (2001). British research on the development of numeracy concepts. In M. Askew & M. Brown (Eds.), *Teaching and learning primary numeracy: Policy, practice and effectiveness. A review of British research for the British Educational Research Association in conjunction with the British Society for Research into Learning of Mathematics* (pp. 10-14). Southwell, Notts: British Educational Research Association (BERA).

Povey, H., Burton, L., Angier, C., & Boylan, M. (1999). Learners as authors in the mathematics classroom. In L. Burton (Ed.), *Learning mathematics: From hierarchies to networks* (pp. 232 - 245). London: Falmer Press.

Prestage, S., & Perks, P. (2001). Models and super-models: Ways of thinking about professional knowledge. In C. Morgan & K. Jones (Eds.), *Research in mathematics education, Volume 3: Papers of the British Society for Research into Learning Mathematics*. London: British Society for Research into Learning Mathematics.

Putnam, R. T., & Borko, H. (2000). What do new views of knowledge and thinking have to say about research on teacher learning? *Educational Researcher, 29*(1), 4 - 15.

Rowland, T., Martyn, S., Barber, P., & Heal, C. (2000). Primary teacher trainees' mathematics subject knowledge and classroom performance. In T. Rowland & C. Morgan (Eds.), *Research in mathematics education Volume 2: Papers of the British Society for Research into Learning Mathematics* (pp. 3-18). London: British Society for Research into Learning Mathematics.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4-14.

Skemp, R. (1976). Relational understanding and instrumental understanding. *Mathematics Teaching, 77*, 20-26.

Spillane, J. P. (1999). External reform initiatives and teachers' efforts to reconstruct their practice: the mediating role of teachers' zones of enactment. *Journal of Curriculum Studies, 31*(2), 143 - 175.

Stein, M. K., Silver, E. A., & Smith, M. S. (1998). Mathematics reform and teacher development: A community of practice perspective. In J. G. Greeno & S. V. Goldman (Eds.), *Thinking practices in mathematics and science learning* (pp. 17-52). Mahwah, NJ: Lawrence Erlbaum Associates.

Tatto, M. T., Schewille, J., Senk, S. L., Schmidt, W. H., Ingvarson, L., Rowley, G., et al. (2006). IEA Teacher Education Study In Mathematics (TEDS-M) conceptual framework: policy, practice and readiness to teach primary and secondary mathematics. Retrieved 30/11/06, from http://teds.educ.msu.edu/documents/IEA-TEDS-M_Conceptual_Framework_September_06.pdf

Teacher Training Agency. (2003). *Qualifying to teach: Handbook of guidance*. London: TTA.

Thompson, A. G., Philipp, R. A., Thompson, P. W., & Boyd, B. A. (1994). Calculational and conceptual orientations in teaching mathematics. In D. B. Aichele & A. F. Coxford (Eds.), *Professional development for teachers of mathematics: The 1994 Yearbook of the National Council of Teachers of Mathematics* (pp. 79-92). Reston, VA: National Council of Teachers of Mathematics.

Wenger, E. (1998). *Communities of practice: Learning, meaning and identity*. Cambridge: Cambridge University Press.