Knowing and Teaching Elementary Mathematics: Teachers' Understanding of Fundamental Mathematics in China and the United States
Liping Ma (1999)

A critical appreciation of the design and conclusions of the Ma study

Ma used the Teacher Education and Learning to Teach (TELT) data base to source the data on American teachers (23=11 +12) in her study. These were described as "better than average". TELT data base was also used by Ball and others.
Ma used same questions with 72 Chinese teachers, who were a "more representative group". This is a small but influential study. The foreword is by Lee Shulman and the acknowledgements include many 'big names' of American [mathematics] education research.
Ma addressed 4 domains of fundamental mathematics /arithmetic:
a) subtraction with renaming, b) multidigit multiplication, c) division by fractions and d) relationship between perimeter and area.

Our focus is on chapter 3, Generating Representations: Division by Fractions.

> Calculate: 1 ¾ divided by ½
> What would you say would be a good story or model for this calculation?

Ma compares the "solid knowledge of the topic" held by the Chinese teachers with the "pseudo conceptual knowledge" which was 'limited and flimsy', exhibited by the American teachers.
There have been similar findings with regard to mathematics subject knowledge of pre-service teachers in UK from SKIMA group and similar findings in Ireland, Turkey (CERME, 2007) and USA (Ball, 1989).

Ma describes a teacher with Profound Understanding of Fundamental Mathematics (PUFM) as not only aware of conceptual structure & basic attitudes of mathematics in elementary mathematics, but also able to teach them to students (p.xxiv).

Critique:
There is another perspective on studying teacher mathematics knowledge in this way: Lave (1988) would describe such a view of cognition as "functionalist". The concept of "knowledge" is tricky. By varying the regime of competence one can identify knowledge or ignorance. At the last seminar we were informed by Hodgen's finding that at least one teacher's SMK was social, situated and distributed. "No cognition without distribution!"
On one hand, Chinese teachers can be seen as communities of practice where high levels of participation and reification complement each other and result in transformative learning, mathematics as meaningful and strong identities as teachers of mathematics (Wenger, 1998).
On the other hand, perhaps the US teachers' "pseudo conceptual knowledge" or understanding of mathematics can be attributed to "an excessive emphasis on formalism without corresponding levels of participation, or conversely a neglect of explanations & formal structure, [which can] can easily result in an experience of meaninglessness." (Wenger, 1998, p.67).

Conclusion from the study:
Teachers will teach better if they are working from a meaningful knowledge base. The interactions between "what it is" and "how to teach it" seem to provide the driving force for the growth of the Chinese teachers' knowledge of school mathematics while collegiality collects momentum for the process. Teaching materials are highly relevant to the process.

Ma recommends changes in a) teacher support; b) teacher education; c) mathematics education research, if American (US) teachers are to develop PUFM.

Issues suggested for attention in the discussion:

1. Use of realistic contexts was often associated with confusion:
   a) confounding division by ½ with division by 2 (10 teachers)
   b) confounding division by ½ with multiplication by ½ (6 teachers)
   c) confuse the three concepts (2 teachers)

How can mathematics educators design for meaningful learning to eliminate this kind of confusion?

2. Dealing with discrepancy:
   - Of 16 with correct or near correct answers
     - 4 did not notice any discrepancy
     - 5 did notice but didn't lead to correct answer
       - 3 doubted possibility of doing it so gave up
       - 1 thought 'real world' thing would be correct
       - 1 explained discrepancy away unsuccessfully (3 and a half quarters are the 'same' as three and a half) Consider the example of Ms Francine (p. 68) in this context.

I see this as an issue for teacher support. What kind of in-service work with teachers could develop the ability to remedy discrepancies? An understanding of usefulness of 'real world' contexts would be helpful.

3. To generate a representation, one should first know what to represent:
I see this as an issue for mathematics education research. Are there enough research based text books available for mathematics teaching? Are mathematics researchers agreed on best representations to use? At a recent mathematics education research seminar in UEA, a representation for teaching an algorithm for division of fractions was offered. It suggested converting both dividend and divisor to same denominator. In this context consider the example of: Ms Felice (p.56) who did just that, but was not deemed confident during computation and did not move to a secure solution.

Dolores Corcoran, St Patrick's College, Dublin City University, Ireland. April 2007