DEVELOPING MATHEMATICS TEACHING AND TEACHERS

A Research Monograph

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CHAPTER 6:

Epilogue

The Epilogue 'looks over' the monograph as a whole to suggest key ideas, critical concepts and theory-practice links. In particular it examines concepts of 'warranted practice', addressing ways in which classroom practices can be grounded and evidenced; 'the pragmatic recontextualisation of theory', to rationalise apparent incommensurabilities in theoretical positions related to teaching, and learning to teach; and 'teacher knowledgeability' to address approaches to teacher learning that avoid deficit models of teachers' knowledge and practice. It goes on to consider methodologies in teaching and teacher education in a discussion of models of practice and their theoretical relatedness. In doing so it makes links to key questions raised in earlier chapters. A concluding section on 'inquiry, development and further research' argues for inquiry-based approaches that avoid deficit perspectives and address the problems and dilemmas of learning teaching; and collaborative research projects between teachers and educators to explore and enhance teaching development for students' more effective learning of mathematics.

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Practice, Theory and Research

Our field is teaching development in mathematics and mathematics teacher education, principally in the UK. We have developed a strong sense of the wealth of practical knowledge rooted in experience that abounds, some of which is reflected in extensive and systematic research, other in small projects and the rest hardly at all in research terms. In a review of recent research in mathematics education, Askew and Wiliam (1995), looking only at research reported in refereed journals, commented how little of the extensive UK research was reported. This perhaps reflects what Cornu (2000), contrasting development in British and French systems of teacher education, refers to as Britain's "pragmatic and empirical culture". Whereas Britain seems to be "increasing still further its heavy emphasis on practical training in schools and de-emphasizing theory, France, which has an *academic* tradition in teacher training is also placing more importance on developing the practical side but within a framework that favours a dynamic interplay between theory and practice" (p. 199; our emphasis). Referring to Cornu, in their editorial introduction, Hoyles, Morgan and Woodhouse (1999) comment on the "refreshing" [for UK readers] acknowledgement by the French of the need to "utilize *collective competence* and take advantage of *research*" (p. 3; our emphasis). One inference is that, in the UK, we need to chart critically our collective competence and make our research more accessible and comprehensive in order to develop possible frameworks in which theory and practice can inter-relate dynamically.

In the same volume (Hoyles, Morgan and Woodhouse, 1999), Ruthven suggests a means to build on the pragmatic and empirical in a "warranted practice" that calls for "evidence over experience", "scrutiny over sentiment" and "argument over advocacy" (p. 212). The term *warranted* has two importance senses: "providing

reasoned grounds for the practice as intended" and "assuring that the practice as implemented does indeed realize its aims" (Ruthven, 1999, p. 210). Together with the suggested inference above, these aspirations offer important ideals for research in our field.

We are concerned here fundamentally with relationships between theory, research and practice. Ruthven refers to McIntyre's (1995) assertion that beginning (and practising) teachers should be encouraged to engage in 'practical theorising' about teaching: "the critical examination, development and experimental use of ideas from many sources, including both the elucidated practice of experienced teachers and also a diverse theoretical and research-based literature" (McIntyre, 1995, quoted in Ruthven, 1999, p. 212). In Chapter 4, we have pointed towards a number of UK programmes or initiatives where such practical theorising is evident: for example in the LAMP/RAMP projects in which practising teachers' collaborative participation in reflective activity in their teaching led to changed perspectives of such teaching; in the Oxford Internship Scheme in which reflective activity and small scale research is built into initial processes of developing teaching; and in small scale programmes of classroom research by practising teachers. However, often, such evidence of practical theorising relating to developments in teaching is seen not to have reciprocal effects in the experience and achievement of students learning mathematics. We can see in these cases evidence of 'reasoned grounds for the practice as intended' but little evidence of 'assuring that the practice as implemented does indeed realize its aims' (Ruthven, ibid).

Ruthven has talked about an "epistemology of professional judgment" (1999, p. 209) which carries with it both strengths and weaknesses. In Chapter 5, we have pointed towards a number of epistemological frameworks in which knowledge is conceived in a variety of terms, each focusing our attention onto particular modes, characteristics or dimensions of human learning and practice. Lerman (2001) speaks of an object of research on mathematics teaching and learning, from a sociocultural perspective, as "a particular moment in the zoom of a lens" (p. 87). The lens metaphor seems a useful one in addressing the layers of complexity here. Elsewhere, Lerman has spoken about a 'gap' between the inter-relation of theoretical and empirical fields in research and their practical manifestations in pedagogical practice (and ultimately official discourse) (reported in Jaworski, 2000a). What we notice all too rarely in the literature are the linkages between different positions on the zoom. Zooming in on pedagogical practice can present a very different orientation to that perceived when the focus is on the epistemological positions of those promoting such practice. It is complex and difficult to seek out these linkages: some researchers/theorisers point to essential incommensurabilities and others to some pragmatic recontextualisation of theory (Confrey, 1995; Jaworski, 2000b; Kieren, Forman and Sfard, 2001). Boaler (2000, p. 5) reminds us of the potential "schism" identified by Alan Schoenfeld in his presidential address to the American Educational Research Association, 1999 "between the 'fundamentally cognitive' and

'fundamentally social' studies of human thought and action" (p. 5). Dwelling on schism, or incommensurability seems to leave us within a realm of esoteric research and theory building that offers little to practitioners and policy makers.

Boaler (2000) introducing an edited book entitled '*Multiple* Perspectives on Mathematics Teaching and Learning' (our emphasis) follows Rogoff (1995) to suggest that "the realisation that learning is at any one time both individual and social requires that previously developed perspectives that give primacy to one or the other need to be adapted or combined" (Boaler, 2000, p. 6). A methodology of pragmatic recontextualisation seems necessary to reach towards such adaptation and explanation for unexplained factors in any theory. Thus pragmatic recontextualisation is a reflexive process that not only uses theory to suggest and promote modes of practice, but through interrogation of practice throws light on problematic aspects of theory. A pragmatic recontextualisation could involve refocusing on theoretical perspectives from the position of dilemmas of practice; for example, a project in Texas has been re-addressing constructivist theory in seeking to explain apparent links between social deprivation and poor achievement in mathematics (Confrey, 2000).

Such recontextualisation of theory is designed to explain the roots of dilemmas and enlighten practitioners in dealing with dilemmas. It points towards the development of methodologies for learning and teaching that are linked clearly to theoretical principles, and through which theories can be challenged. However, there are issues and dilemmas arising from practical implementation of theoretical ideas. For example, situated learning and a focus on language has been interpreted as requiring methods of group or project work; constructivist theory has been interpreted as requiring methods to promote cognitive conflict. While it is possible to make clear the theoretical links between these theories and methods, a danger is that the methods and associated practices assume regulatory mechanisms whose meanings and purposes are unclear, and sometimes damaging, to the learners they control (Boaler, 2000). For example, Halai (2001) shows how the well-intentioned use of documented strategies of *cooperative learning* in a mathematics lesson in Pakistan resulted in the alienation of one pupil whose fear of 'reporting-back' prevented his engagement with the mathematics of the activity.

This example highlights a number of kinds of issues related to the development of *warranted practice* (Ruthven, 1999) We see a problematic interface between Ruthven's principles of "providing reasoned grounds for the practice as intended" (in this case strategies of cooperative learning) and "assuring that the practice as implemented does indeed realize its aims". One set of issues is related to ways in which theories, such as that of cooperative learning, are interpreted and realised in mathematics classrooms; another set concerns teachers' understandings of pedagogic approaches, such as those *related* to cooperative learning, and their relation to principled learning of mathematics; yet a third set concerns ways in which *learning* is conceptualised as in 'cooperative learning' or in 'principled learning of mathematics'. These issues highlight the complexity of knowledge required of teachers in striving

for effective learning situations.

Teachers' Knowledge and Knowledgeability

Chapter 2 has identified a range of elements of teachers' knowledge and has, in particular, problematized the relationship between content, or subject matter knowledge (in our case, *mathematical* knowledge) and pedagogical knowledge. The three questions raised at the end of Chapter 2 highlight the problems in teachers' development of mathematical knowledge, particularly syntactic knowledge; how university mathematics courses and teacher education courses do or can contribute to this development, and whether mathematical and developmental needs are different for primary and secondary teachers. The transformation of mathematical knowledge from 'learner knowledge' to 'teacher knowledge' (Prestage and Perks, 1999a) seems an important step in a teachers' growth of knowledge and in the development of teachers' *knowledgeability* (Lave, 1993; Adler, 2000).

Askew and Wiliam (1995), after reviewing research related to pupils' learning and related classroom approaches, state that "Many aspects of mathematics teaching are under-researched'. They point to the importance of focusing research on "the strategies which might be offered to teachers to affect their practice" (pp. 42-3). However, more importantly – as illustrated by the example from Halai above - seems to be how teachers develop knowledgeability of such strategies; that is principled understandings of the nature and purpose of such strategies that translates into classroom practice. Kowledgeability involves teachers in developing their ability to relate mathematical knowledge to such knowledge of strategies in ways that lead to effective mathematical learning of pupils in classrooms.

The concept of knowledgeability recognises overtly the deficit discourse we have mentioned in earlier chapters whereby through certain ways of expressing the knowledge that is expected of teachers (and indeed their students) we come to talk about what is lacking or missing and a concomitant necessity for remediation. Official rhetoric, seeking to improve educational outcomes, often contributes to the problem.

Adler (2000) introduces knowledgeability as follows:

> I use the concept of "knowledgeability" to capture and produce a dynamic and situated notion of learning, of coming to know about teaching. … . Knowledgeability contains within it a sense of being knowledge-able. As a positive statement, I believe, it provides a conceptual tool that could assist us to shift away from a deficit discourse in teacher professional development. (p. 33)

Adler's notion of situated learning accords with theories of communities of practice, Wenger's (1998) "process of becoming" and much current writing about teaching development that is rooted in social theories and referenced in Chapter 5. Teachers are seen to belong to multiple intersecting communities involving "social, political, economic, cultural and historical contexts" (Adler, 2000, p. 33).

Research needs to address how knowledgeability develops, from both situated and cognitivist perspectives, and what are the factors for teachers either as individuals or members of intersecting communities that contribute to knowledgeability. For example, how does a shift from knowing mathematics as a learner to knowing as a teacher develop? How is such a transformation related to development of pedagogy, and to mathematical learning in classrooms? These might be seen as questions relating to individual and cognitivist positions. On the other hand we might ask how communities of learners and of teachers of mathematics intersect. What are the nature and practices of such communities? How are teachers as learners of mathematics constituted and how do they relate to students as learners of mathematics?

Questions such of these have rhetorical force but little practical force. In their current form they are not researchable questions. To arrive at researchable questions a clear account of context, conditions and constraints is necessary – as in Confrey's research in Texas, or Halai's in Pakistan classrooms, or with teachers in the UK struggling to implement a highly specified numeracy curriculum. As we struggle with questions about teacher learning and its promotion we also face philosophical and epistemological issues in considering development of teaching knowledge. How to deal with dilemmas highlighted in Chapter 3, 4 and 5 is a part of this struggle. What kind(s) of research will address such questions and how can we arrive at questions which are researchable but not reductionist: i.e. which allow us to design specific lines of action within the complexities we recognise?

The Theoretical Basis of Methodology

Conclusions from Chapter 2 about the knowledge of teachers for teaching mathematics included the following:

a. formal qualifications are not reliable indicators of effective mathematics teaching (in the primary years);

b. 'secure' knowledge of mathematics (its modes of inquiry and the integrity or 'connectedness' of its content) is clearly associated with primary mathematics teaching judged to be effective;

c. there is little evidence to suggest that teachers' knowledge of mathematics develops as a consequence of teaching.

One inference, or proposition, from these statements combined is that formal qualifications do not ensure 'secure' knowledge of mathematics, and moreover, such secure knowledge of mathematics does not develop as a result of engagement in teaching mathematics. These are very interesting, although salutary, propositions. They offer further hypotheses or inferences leading to questions for future research, such as:

1. How can programmes leading to formal mathematical qualifications take account of findings regarding the nature of 'secure' mathematical knowledge, seen to be so valuable for teaching?

2. In what ways throughout training and subsequent practice in teaching, can teachers' knowledge of mathematics develop 'secure' forms?
3. How can we develop clearer theoretical and practical understandings of 'secure' mathematical knowledge and its relationship to pedagogy?

The idea of *secure* mathematical knowledge (or knowledge of mathematics) has currently a popular basis that is research related. It is also fundamentally *theory* rooted. Its exploration would be hollow without a clear epistemological analysis. Thus, the operationalisation of knowledge needs to be linked to the social epistemology of mathematics from which it is operationalised. From such considerations, a methodology of operationalisation can be sought; e.g. models for the development of secure forms of knowledge. Although use of such language makes the process seem technocratic, it is actually deeply social and developmental. It draws on all the theoretical positions outlined in Chapter 5, and challenges incommensurability or schism.

Experience and research has shown us that transmission models of knowledge transfer do not work; cascade models are one prime example. We might therefore look to models that are rooted in the theories outlined in Chapter 5. A danger as we have seen lies in trying to convert such theoretical perspectives into (simple) methodologies. Complexities of "multiple intersecting communities involving social, political, economic, cultural and historical contexts" (Adler, 2000, p. 33) have to be acknowledged and addressed with concomitant avoidance of pathologising particular groups of learners (e.g. with relation to race, gender or learning difficulty) (Adler and Lerman, 2001). The zoom of the research lens will inevitably focus attention depending on what is asked and how it is asked. The theoretical and practical implications of what is asked, and how, have to be acknowledged critically, as does, importantly, what is on the periphery of the zoom. This takes seriously Corbin's words relating to the incomplete nature of models of the knowledge required for teaching mathematics, quoted in Chapter 2: "part of their usefulness can be at their borders, in what they specifically exclude and include in particular instances of their use" (2000, p. 2).

Chapters 3 and 4 have made clear the proliferation of models in teacher education at both pre-service and in-service levels. The extent to which these models are research based, or theory explicit varies considerably. Although we can argue over their research-based nature, for example whether they rest on academically rigorous research or 'popular' research (as mentioned earlier), there is little doubt that few of them make explicit their theoretical/epistemological roots. These recognitions lead to yet more questions. Two questions expressed in Chapter 3 were:

- How can students [pre-service teachers] develop a capacity for working on their own professional development in a way that relates to their personal aspirations of what it is to be a teacher?
- How can we as educators model such activity as we balance the competing

demands of individual professional autonomy and collective state and student entitlement?

Asking 'how' leads to considerations of methodology. Our discussion here suggests that before we can address these methodological questions, and move towards models, we have to make explicit the theoretical perspectives that underpin the 'how' questions. For example, we can see individual or cognitivist elements in these questions (e.g., working on their own professional development; personal aspirations; individual autonomy) and sociocultural elements (e.g., pre-service teachers' [collective] professional development; collective entitlement). These terms need an epistemological analysis relating to mathematical and pedagogical knowledge and its social rooting in practices and relationships before research can be originated or models conceptualised. For example, the Japanese model, discussed briefly at the end of Chapter 4, seems to embody a number of these elements, such as individual or collective professional development; it also demonstrates teachers' clear focus on the mathematics to which teaching is related. We can analyse this model relating its particular characteristics to theoretical perspectives that explain and justify its approaches, while learning from the dilemmas that arise in its implementation. It is only by insisting on such epistemological rigour that is more than tokenistic *at the beginning* of the methodological process that models can be effectively scrutinised and evaluated. In all of this process, we need to keep in mind that our field lies within *mathematics* education, and our talk about models of professional practice such as 'partnership' and 'mentoring' must be firmly rooted in mathematical epistemology and well as social epistemology more generally.

Inquiry, Development and Further Research: Conclusions

In drawing this monograph to a close we shift from consideration of big ideas such as warranted practice, pragmatic recontextualisation of theory, and teacher knowledgeability, to a conclusion, a 'thesis' drawing on all our chapters and sections above. The teacher/teaching relationship was highlighted in the Prologue. While we might see ITT and CPD to be largely about teach*ers* and their development (both of which lead to teach*ing* development of sorts), DCI seems to be more explicitly about developing teach*ing*. Thus, while models of developing teachers can lead to a deficit discourse, we look to DCI models to offer an alternative approach to addressing the tensions, constraints and difficulties in teaching. So rather than looking at what is wrong with teaching, possibly in terms of a lack of evidence of students' 'effective' learning of mathematics, we explore together (where 'we' includes teachers and academic researchers) the issues of teaching and students' learning.

As we seek appropriate methodologies from which models can be formulated we need to make overt the theoretical/epistemological foundations on which we build. This itself involves inquiry (or meta-inquiry) and we have to take care not to engage in an endless epistemological spiral. There is considerable evidence, much of it very localised or small-scale, of genuine collaboration between teachers and educators in a spirit of inquiry, leading to development in positive directions (regarding students'

learning). The participants in such inquiry are likely to have very different experiences, knowledge, interests, strengths as well as roles and goals in the collaborative enterprise. In order to work productively for development, knowledge needs to be contributed according to its focuses and strengths. Thus, for example, the epistemological foundations of a model or project would most likely be the responsibility of those confident in such knowledge, whereas knowledge of the social setting, its complexity of inter-relationships might come from other partners. However, for effective outcomes, it seems to us that critical negotiation of all layers of knowledge, experience and decision-making are crucial. We do not underestimate the complexity and the problems that this poses at a variety of levels. However, over-simplification can lead not only to limited outcomes, but also to developmental disappointment and frustration.

To some extent we are arguing for projects of large scale and scope, and recognise those that are taking place currently in the second phase of the Economic and Social Research Council's (ESRC: www.esrc.ac.uk/) Teaching and Learning Research Programme (TLRP: www.tlrp.org/). None of the projects involved are overtly about mathematics, although mathematics learning and teaching are significant in at least three of them. Gaining large scale research funding at a national level is highly competitive, and it is likely that projects that could be hugely beneficial in the terms we have set out do not receive the necessary funding. Thus it is important not to undervalue the smaller studies that have been undertaken, and are being undertaken, but to recognise that these individually have less impact. Meta-analyses can help, and should be done more extensively. However, people initiating research at whatever level need to think clearly about potential developmental impact as well as esoteric contribution to knowledge. For example, those supervising doctoral research have a responsibility to their students and the wider community that the results of three years or more of intensive study should be known and used more widely than just the local research group.

While saying this, we recognise the movement towards evidence-based practice and the roles of classroom teachers engaging in research. Inevitably much of this work is extremely localised, and although funding providers encourage a variety of forms of dissemination, this itself needs scrutiny if it is to be an effective means of communication in terms of promoting teaching development through negotiative dialogue. The requirement for teacher research grants in the UK to be linked to a Higher Education Institution encourages such negotiative dialogue. As partners, the teachers, school and HEI academics have potential to achieve more than any one of them can independently. A basis of collaborative inquiry again offers epistemological coherence.

And so we arrive at and conclude with a statement of our thesis. We are arguing for inquiry-based research/development involving genuine collaboration and power sharing between teacher researchers and academic researchers in mathematics education with clearly defined goals directed at teaching development in order to

improve student learning, and with clearly articulated epistemological bases. All concerned have the important task of convincing funders and educational policy-makers that this approach has far-reaching potential to achieve the effective education of students in mathematics.