DEVELOPING MATHEMATICS TEACHING AND TEACHERS

A Research Monograph

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CHAPTER 2:

Subject And Pedagogical Content Knowledge For Teaching Mathematics

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Introduction

In recent years official concern over standards of mathematics in England has been driven by two factors. First, a substantial amount of international pupil performance data has become available (e.g. Reynolds and Farrell, 1996), including data from high profile comparative studies such as TIMSS (e.g. Harris, Keys and Fernandes, 1997), which has focused attention on the relatively poor mathematical attainment of British pupils. Secondly, a burgeoning national audit culture of league tables, targets, and inspection evidence has focused attention upon the variation in standards nationally. During this period much of the educational policy and prescription emanating from government bodies has been premised upon the belief that improved pupil performance will be achieved through improving teachers' subject knowledge. This is evident in the close prescription of subject knowledge in the National Curriculum for ITT (DfEE, 1998a), and the requirement for an audit of that knowledge. Additionally, all student teachers are now required to take, and eventually pass, a national 'numeracy skills' test in order to achieve Qualified Teacher Status. Successive governments, over the last decade, have moved towards an increasing level of prescription from "the what of the curriculum content to the how of teaching methods" (OFSTED, 1995, p. 8 quoted in Brown *et al*., 1999). This policy has recently resulted in the implementation of the National Numeracy Strategy (DfEE, 1999b) and a cascade training programme for all primary teachers [pupil age 4/5-11] and Key Stage 3 [age 11-14] teachers.

This chapter includes accounts of studies of the mathematics content and pedagogical knowledge of pre-service as well as serving teachers. This material has been included here, rather than in the next chapter, because the major issue here is teacher knowledge rather than the particular circumstances of initial teacher education (the focus of Chapter 3).

Conceptualising Teacher Knowledge

A recurrent theme in the Prologue to this monograph was the diversity of kinds of knowledge that underpin teachers' preparation for teaching and their subsequent actions in the classroom. Debate about the role of subject knowledge in teaching has spanned the 20th century. Dewey (1904) held that knowledge of subjects included knowledge of inquiry in the particular domain(s), and therefore knowledge of teaching method as he conceived it. Such a view is consistent with that held in the medieval universities, where no distinction was made between knowledge of content and knowledge of how to teach it. Indeed, the modern form of doctoral examination (defence of the thesis) originates in a medieval *inceptio* which was based on the belief that understanding was *demonstrated* by the act of teaching (Shulman, 1986). The philosopher John Wilson (1975) similarly held that comprehension of the logic of concepts offered guidance on how to teach them. In recent years the importance of subject knowledge has been well documented and the lack of it linked to less competent teaching (Wragg, Bennett and Carré, 1989; Bennet and Turner-Bisset, 1993; Simon and Brown, 1996; DES, 1983; OFSTED, 1994; Rowland, Martyn, Barber and Heal, 2000, 2001) and over-reliance on commercial schemes (Millett and Johnson, 1996).

There is a tension in the literature between those who pay little or no attention to teachers' subject knowledge *per se*, those who consider it in isolation, and those who portray the 'mathematical knowledge' necessary to teach 'effectively' as more complex than simply requiring a grasp of the relevant mathematics subject knowledge. During the early 1980s qualitative research based in classrooms explored the organisation of subject knowledge in teaching, but scant attention was paid to how subject knowledge was integrated into planning and classroom actions. Thus, Leinhardt and Smith (1985, p. 8) wrote: "No one asked how subject matter was transformed from the knowledge of the teacher into the content of instruction". The vital component of the complex teaching nexus is now conjectured to be this transformation of mathematical content, or subject matter knowledge (SMK) into a form appropriate to teach. The term 'pedagogical content knowledge' (PCK) was first employed by Shulman (1986) to depict "the particular form of content knowledge that embodies the aspects of content most germane to its teachability" (*ibid*., p. 9). This pedagogical repackaging of mathematics necessitates facility with the representations, illustrations, examples, explanations and analogies to make mathematical ideas comprehensible to others.

The notion that the development of PCK is the most important element of teacher knowledge to underpin teaching (including elementary school teaching) is currently popular both in Britain and America. Shulman (1986) argued that too much emphasis had been placed in recent (American) research on general pedagogical processes and not enough on subject matter knowledge, attention to which he called "the missing paradigm". In a seminal text, he subsequently delineated seven categories of teacher knowledge:

- content knowledge - both 'substantive' and 'syntactic' (see below);
- general pedagogical knowledge - generic principles of classroom management;
- curriculum knowledge - materials and programmes;

- pedagogical content knowledge - which for a given subject area includes forms of representation of concepts, useful analogies, examples, demonstrations;
- knowledge of learners;
- knowledge of educational contexts, communities and cultures;
- knowledge of educational purposes and values.

(Shulman, 1987, p. 8; see also Wilson, Shulman and Richert, 1987, p. 113)

Bennett and Carré (1993) observed that the relationship between these bases, and the nature of their connection with classroom performance, was by no means clear: they nevertheless affirmed the framework as a "useful starting point in conceptualising students' learning to teach" (p. 7). Even and Tirosh (1995) draw attention to the lack of evidence to support and illustrate the supposed interrelation between SMK and PCK, and suggest that this might, in part, be due to different conceptions of the role of the teacher. At one extreme, the teacher is viewed as the manager of an expert-made curriculum and teacher-proof materials. At the other, s/he is is more engaged with the subject matter itself and ways of representing it in appropriate ways. Even and Tirosh add the telling comment that "research on learning and learners, and research on teaching and teachers have been following separate tracks for a long time" (p. 3). Their research demonstrates the relationship (for a group of Israeli secondary mathematics teachers) between SMK and knowledge of students' common conceptions and ways of thinking on the one hand, and their presentation of subject matter and response to students' ideas and questions on the other.

Critics, however, claim the framework to be not sufficiently dynamic to allow for a non-absolutist view of mathematics (Meredith, 1995), decontextualised (Stones, 1992) and presenting a simple transmission view of teaching (Meredith, 1993; McNamara, 1991; McEwan and Bull, 1991). McNamara (1991) questions whether the distinction between content knowledge and pedagogic knowledge can and should be made, since all mathematics subject matter is itself a form of representation.

These critiques must be evaluated against Shulman's (1987) own clear admission that his framework was provisional, tentative and most probably incomplete. There is, we would suggest, an undeniable sharpness of insight in Shulman's analysis – a blend of philosophy and empirical reasoning – that transcends the critique. A more recent elaboration and refinement of Shulman's scheme, explicitly related to current developments in ITT in the UK, can be found in Turner-Bisset (1999).

The focus of the present chapter is on what Shulman called "the knowledge that grows in the minds of teachers [of mathematics], with special emphasis on content." (1986, p. 9). These correspond to the first, third and fourth of Shulman's seven categories listed above. The plethora of similar but not-quite-identical names for similar concepts is a potential source of confusion here, not least for the author himself. In a footnote, Shulman (1987, p. 8) writes, "I have attempted this list in other publications, though, admittedly, not with great cross-article consistency".

Grossman, Wilson and Shulman (1989), drawing on data from the Stanford University *Knowledge Growth in a Profession* programme, propose three dimensions important to the task of teaching: factual information, central concepts and organising principles of the discipline. Content knowledge (they argue) would appear to emerge through a process of critical analysis, which is guided by the substantive and syntactic structures of a discipline. Broadly speaking, substantive knowledge can be characterised as including knowledge of facts and concepts, and the ways that they are organised. Syntactic knowledge is about the nature of inquiry in the field, and the mechanisms through which new knowledge is introduced and accepted in that community; it includes knowledge about proofs and rules of structures (Schwab, 1978). Such dimensions, implicit in the undergraduate learning of mathematics must, it is conjectured, be rendered explicit in order to teach mathematics. Knowledge of substantive and syntactic structures has implications for what teachers choose to teach, and how they teach (Shulman, 1986, p. 9). The emphasis for Shulman is on key problematic moments of 'contradiction' and 'incompatibility', on transition and process as well as a relational view of knowledge (Corbin, 2000). Shulman also distinguishes between two kinds of understanding: knowing 'that' and knowing 'why': "The teacher need not only understand that something is so; the teacher must further understand why it is so" (1986, p. 9). Skemp (1976) also provided powerful arguments for this position in his classic paper on instrumental and relational knowledge in mathematics.

Feiman-Nemser and Buchmann (1985) introduce the concept of self as they characterise 'pedagogical thinking' in terms of 'strategic, imaginative, and grounded in knowledge of self, children and subject matter'. Askew, Brown, Rhodes, Johnson and Wiliam (1997a) identify as a framework for analysis of effective teacher knowledge and practices: teacher beliefs, pupil responses and teacher pedagogic content knowledge (dependent in turn upon knowledge of teaching approaches, knowledge of pupils and subject knowledge). Indeed, Askew (1999) argues that all propositional statements (beliefs, concepts, knowledge) are constructed through discourse and hence delineation can only be established by social definition. There is evidence (Brown *et al*., 1999) to indicate that such a 'playing down' of the image of 'hard' knowledge is useful and comforting for primary teachers in the process of change. Brown *et al.* (1999) also suggest the need to reconcile the dualities of phenomenological and hard-edged, official versions of mathematics. Such observations implicitly recognise the philosophical differences underpinning different perspectives on mathematics, and their influence on the ways that teachers think about mathematics (e.g. Sanders, 1994).

Teachers' Knowledge in Action

Much research into the area of subject knowledge and pedagogical content knowledge (e.g. Shulman, 1986; Wilson *et al*. 1987; Tamir 1988; Aubrey 1997) focuses upon the key notion of the *transformation* of subject matter knowledge for the classroom, in relation to teachers' knowledge about explanations, tasks and activities, and styles of teaching and learning. Shulman reports an episode where, in his estimation, lack of content knowledge was the underlying reason for less effective teaching (1987, pp. 17-18). The Stanford programme shows teachers using various coping strategies when they lack subject matter knowledge, including relying heavily on 'the textbook' (*c.f.* Millett and Johnson, 1996, pp. 54-74) and avoiding discussion and questions. One case study contrasts Joe, who had been a doctoral mathematics student, with Laura, a 'non-math major'; Joe allowed students to generate and evaluate their own methods for solving problems, whereas Laura 'drilled' students in the book-algorithm and was reluctant for them to use any other methods (Steinberg, Haymore and Marks, 1985). Later Shulman and others working in this area (Wilson *et al*., 1987) developed 'intellectual histories' through a series of semi-structured interviews and outlined a six stage model of pedagogical reasoning in action: comprehension, transformation, instruction, evaluation, reflection, new comprehension (Wilson *et al*. 1987, p. 119). They claim that making the transition from novice to expert involves developing many different representations of the same knowledge.

The work of Deborah Ball on the mathematics knowledge of prospective teachers has been influential in the USA and beyond. Ball echoes Shulman's constructs of substantive and syntactic knowledge in any discipline by making a distinction between knowledge *of* mathematics (meanings and underlying procedures) and knowledge *about* mathematics (what makes something true or reasonable in mathematics). Investigating both elementary and secondary pre-service teachers' understanding of division, she found (Ball, 1990a) that both had significant difficulties with the *meaning* of division by fractions. Most could do the calculations, but their explanations were rule-bound, with a reliance on memorising rather than conceptual understanding. They believed that mathematics *could* be meaningful but lacked the knowledge of these meanings themselves. Ball argues that not only should mathematics be revisited in initial training, but also that pre-service teachers may also need to 'unlearn' what they know and believe about the teaching and learning of mathematics.

In an earlier study, Ball (1988) explored the distinction between what mathematics should be known and how it should be organized. The 'what' tends to be check-listed, as in the regulations for ITT (DfEE, 1998a), whereas the 'how' is typically described qualitatively by words such as 'flexibly', or 'in-depth'. Her term 'connected' (Ball, 1990b), derived from comparison of the knowledge held by expert and novice teachers, has since been used by others as a way of describing the quality of subject knowledge that teachers need (e.g. Askew *et al*., 1997a).

In the UK, Aubrey (1993, 1994a, 1994b, 1995, 1996, 1997) observed classroom processes to unfold a tripartite relationship between children's informal mathematical knowledge; teachers' subject and curriculum knowledge, values and beliefs; and classroom practice (exemplifying in her view pedagogic subject knowledge). In an analysis of the mathematical pedagogical subject knowledge of four experienced 'reception' class [age 4-5] teachers, Aubrey illustrates the co-ordination and use of teacher and pupil knowledge in the complexity of ongoing classroom processes (Aubrey, 1997). She concludes that subject content knowledge has a crucial effect on pedagogical practice "even at this early stage of schooling" (p. 144). In the case of one teacher secure knowledge of content and pedagogy gave her confidence to set up explorations, bring out mathematical relationships and display them in various ways; whereas, another teacher lacked a firm grasp of subject matter and was unable to develop explanations or questions effectively. Aubrey emphasises the centrality of subject content knowledge and knowledge of pupil competence to the teacher's pedagogical subject knowledge. This dynamic process of knowledge development requires teachers to have a rich and deep understanding of the main conceptual fields, as well as a grasp of their interconnectedness.

Prestage and Perks (1999a) question the assumption that teachers have full access to subject matter knowledge. They argue that for both experienced and novice teachers much subject matter knowledge remains as 'learner-knowledge' and is not transformed into 'teacher-knowledge' (a case study of 'Frances' in Rowland *et al*., 2000 also describes conditions under which this transformation did and did not – for her – take place). The capacity to transform personal understanding was posited by Prestage and Perks to depend on what teachers brought to the classroom; many experienced teachers used their own mathematical experiences as a pupil in the classroom as a foundation for making decisions (Prestage, 1999). There was little evidence to suggest that teachers' subject matter knowledge developed as a consequence of teaching.

Indeed, Grossman *et al*. (1989) had also argued that teacher educators ought to consider how best to introduce the teaching of subject matter knowledge into initial teacher education, since it could not be assumed that this knowledge would have been acquired through undergraduate study in other departments. Their findings, supported by Aubrey (1997), give yet further evidence that while knowledge of learning, teaching and classrooms increases with experience, knowledge of subject content does not. The situational nature of teachers' decisions and rationalisations about classroom tasks was the focus of the MAST project in which Simon and Brown (1996, 1997) found that "different ways of developing teachers' subject knowledge need to be investigated and evaluated" (1997, p. 7). There is evidence to suggest that teachers need to work in ways that relate subject knowledge and pedagogy at the level they will teach. In the United States, for example, Schifter (2001) has worked with teachers on the subject knowledge relevant to their classroom teaching, using methods directly applicable to ways they might work with students.

Predictors of Teacher 'Effectiveness'

In a review of USA research dating from the 1970s, Grossman *et al*. (1989) found no statistical significance regarding the relation of subject knowledge to effective teaching (as measured by 'student achievement'). Wilson *et al*. (1987), reviewing research from these early studies, also found that they failed to yield consistent findings which correlated teacher knowledge with student achievement: thus more teacher subject knowledge did not necessarily mean better teaching. Byrne (1983) suggested that one reason why these earlier studies failed to establish a relationship between teacher subject matter knowledge and student achievement is that both notions had been inadequately conceptualised and measured - the first, typically, by multiple-choice tests and quantity of tuition received, the second by standardised tests. Research by Begle (1968, 1979), and later Ball (1990b), also challenged the assumption that the more subject knowledge teachers had (as measured by quantity of instruction received) the more 'effective' they were. Indeed, they questioned whether some aspects of higher level mathematics training might actually be counterproductive in preparation for teaching.

In a recent study of primary teachers in the UK, Askew *et al*. (1997a, p. 65) also found that 'more' was not necessarily 'better' when they correlated the teachers' mathematical knowledge, measured in terms of qualifications, against pupil learning over one academic year. However, the study found that those teachers who had knowledge and awareness of conceptual connections within the primary mathematics curriculum were likely to be more effective (as measured by average pupil test gains). The study cohort of 90 teachers included three with mathematics degrees and a further 10 who had studied A-level mathematics, of whom 8 had passed. The median class gain was slightly lower for the teachers who had studied A level compared with the other teachers. Thus there was a slight, but not statistically significant, negative correlation between level of mathematical qualification and 'effective' teaching: indicating that mathematical qualifications are not (in isolation) a reliable indicator of the mathematical knowledge required for teaching. One graduate was reported to be unable to explain the links between elementary concepts such as fractions and decimals. In fact, the amount of extended in-service, continuing professional development in mathematics education (such as '20-day' courses: see Chapter 4 for details) undertaken by the teachers was found to be a better predictor of 'effectiveness' than formal qualifications (Askew *et al*., 1997a, pp. 74-79).

International comparisons of pupil performance yield much interesting data upon which to speculate by way of explanation. Chinese pupils, for example, outperform their USA contemporaries. Ma (1999) contrasted the mathematics content knowledge and PCK of USA elementary school teachers with their counterparts in China. She found that the knowledge of the USA teachers studied was relatively instrumental, unconnected and devoid of conceptual grounding. The Chinese teachers, with fewer years of formal education and inferior mathematical qualifications, had acquired a strong conceptual grounding in mathematics (which Ma calls 'profound understanding of fundamental mathematics') that influenced the ways in which they worked with children.

Pre-Service Teachers' Mathematics Content Knowledge

Mathematics and English components of primary teacher training courses have been judged to be amongst the most satisfactory when compared with other subjects (DES, 1991b). Nevertheless, the change in subject matter knowledge of mathematics of primary student teachers during training was found to be not significant by Carré and Ernest (1993) in a Leverhulme-funded study of 59 primary Postgraduate Certificate in Education (PGCE) trainees who chose to specialise in one of mathematics, science, music and 'early years' (Bennett and Carré, 1993). Instruments developed for the assessment of knowledge for teaching mathematics – drawing on Shulman's categories - took into account students' beliefs about mathematics, in addition to content knowledge and pedagogical application. Higher order aspects of substantive and syntactic knowledge were assessed, as were students' attitudes (interest and confidence) towards mathematics – a teacher characteristic identified by Ernest (1989) as significant, yet absent from Shulman's analysis. The mathematics and science specialists performed significantly better than the music and early years groups in the pre-tests of content knowledge; the syntactic knowledge of mathematics specialists was dramatically superior to that of the other three groups in an item about the strategies needed to solve a problem. In the post-test, the music specialists showed the greatest gains in content knowledge for teaching, although their improvements were in content and pedagogical application and not in higher level knowledge of structure and inquiry. The study also compared the teaching of mathematics, science and music by the 'specialists', in comparison with the whole cohort. When classroom teaching performance was assessed, it transpired that there was "virtually nothing to distinguish mathematicians and others in teaching mathematics" (p. 161). The situation was very different in the teaching of music, where specialists were judged to perform at a higher level of competence than other students. This was in accordance with the researchers' expectations (p. 164), since music specialist students had been assessed as having a high level of music subject-matter knowledge relative to other students, while this was not the case for mathematics specialists *vis a vis* mathematics.

The relation between subject knowledge and 'effective' teaching was investigated by Rowland, Martyn, Barber and Heal (2000) in a study of the audit and remediation of 154 primary pre-service (PGCE) teachers at one University in the academic year 1997-98. A 16-item test was administered to them some four months into their one-year course, after the main content areas had been 'covered' but before the first of two extended school-based placements. Items on generalisation and proof were found to be among the most demanding. On the basis of this audit, the level of each student's subject knowledge was categorised as low, medium or high, corresponding to the need for significant remedial support, modest support (or self-remediation), or none. On the final school placement, specific assessments of the students' teaching of number were made (against the 'standards' set out in DfEE, 1998a) on a three-point scale weak/capable/strong. There was an association between mathematics subject knowledge (as assessed by the audit) and competence in teaching number. Further analysis identified that students obtaining high (or even middle) scores on the audit are significantly more likely to be assessed as strong numeracy teachers than those with low scores; students with low audit scores are more likely than other students to be assessed as weak numeracy teachers. In a follow-up study, data for the subsequent (and larger) 1998-99 cohort were subjected to similar analysis, with similar findings. Following Bennett and Turner-Bisset (1993), more extensive data from school placements enabled comparison of mathematics subject knowledge with teaching performance on both first and second placements also with respect to both 'preactive' (related to planning and self-evaluation) and 'interactive' (related to the management of the lesson in progress) aspects of mathematics teaching. The association between audit score and teaching performance was significant ($p<5\%$) in three of the four analyses (Rowland *et al*., 2001), the exception being the first placement/preactive analysis (with $p=8\%$). It should be noted that the mathematics subject knowledge of these students was determined by means of an instrument designed for the purpose, and this instrument (as opposed to formal qualifications gained sometime in the past) provided the three subject matter knowledge categories for comparison with classroom performance. Indeed, there was again no simple relationship between level of formal qualification in mathematics (gained several years earlier) and the audited level of mathematics subject knowledge. The 27 (out of 154) students with an 'A' level pass were distributed throughout the 'top' two-thirds on the audit. An independent statistical analysis (Proctor, 2001) of the 1998-99 data incorporates hitherto unexamined variables in an attempt to 'predict' the mathematics teaching competence of these trainees (see also Rowland, 2001). These additional variables are the gender of the trainee, their chosen age specialism (Early Years 3-8 or Middle Years 7-11), and a subject knowledge confidence self-assessment incorporated into the audit. Proctor observes that although there is no significant gender factor in the audited subject knowledge, the males were more confident (as a group) than the females, the female/Early Years students being the least confident group. As a group, females were more competent teachers of mathematics than the males. Regarding teaching competence, the predictive effect of the self-assessment score is negligible for the Middle Years group compared with that of the audit score and the gender effect, whereas for the Early Years trainees confidence is the more powerful predictor.

At another UK university, Goulding and Suggate (2001) also conducted an analysis of the responses of 201 primary student teachers' to an audit of their subject knowledge. They reported errors on questions in the following domains: appropriate degrees of accuracy in area calculations (84% making errors); proof (61%); calculations involving volume and mass and density (45%); ordering small numbers (44%). They conjectured that problems with measurement may be to do with assessing, estimating and approximating strategies in a written test, although they may also reveal problems with the idea of a continuous number line, and the relationship between mass, volume and density. Ordering small numbers threw up difficulties with ordering when some of the numbers had three or more decimal digits. Most student difficulties seemed to respond to one-to-one remediation, but their problems with proof seem to be of a different nature, being much more difficult to remedy. Goulding and Suggate conjectured that deductive proof was much more problematic than proof by exhaustion or disproof by contradiction. The proof question also threw up fear of and difficulties with algebra, which were not highlighted to such a degree in other algebra questions on the paper. Rowland *et al*. (2001) also identify aspects of proof as the source of acute difficulty in a cohort of 173 primary trainees, and an apparent reluctance to generate examples when deciding whether a general statement is true or false. The evidence, in the UK, points to the conclusion that deficiencies in syntactic knowledge are especially resistant to remediation in the primary PGCE training year, and systems of support for teachers need to look well beyond pre-service training in this respect.

Subject Knowledge in Relation to Beliefs and Attitudes

There is compelling evidence to suggest that experiences as a learner of mathematics, conceptions about the nature of mathematics and instructional practices as a teacher of mathematics are all profoundly interconnected (Thompson, 1984, 1992; Lerman, 1986, 1990; Lampert 1988; Meredith 1993; Sanders, 1994). Beliefs about the nature of mathematics have been found to be "not inconsistent" with dominant pedagogic beliefs (Andrews and Hatch, 1999); and, to play a significant role in shaping teacher behaviours (Askew *et al*., 1997a; Lerman 1986, 1990; Ernest, 1989). Bibby (1999) charts the emotional aspects of primary teachers' mathematical subject knowledge and argues that emotion is a powerful element of change both in positive and negative ways.

A number of studies have investigated the beliefs and attitudes of student teachers. Meredith (1993), for example, in a study of 12 students taken from across three different courses, concluded that students' views about pedagogic content knowledge and their learning of it was not robustly connected to their training. Rather, she concluded, differences in attitudes may have been due to prior learning, knowledge, experience, values and epistemological beliefs. See also Aubrey (e.g. 1997); Carter (1990); Lave and Wenger (1991); McNamara (1991).

Andrews and Hatch (1999) conducted a factor analysis of the conceptions and beliefs about mathematics and its teaching of 577 secondary teachers of mathematics across 200 schools. The study identified five different conceptions of mathematics and five of mathematics teaching. Although dominant conceptions of mathematics manifested in (largely) commensurate beliefs about teaching, the authors found the teachers often possessed disparate conceptions simultaneously and argued that this may be a consequence of "cultural and curricular ambiguities in respect of mathematics teaching in England" (p. 203).

The negative attitude of some secondary mathematics PGCE students towards their experience of learning mathematics as a subject at undergraduate level is evident from some recent work of Anderson, Goulding, Hatch, Love, Morgan, Rodd and Shiu (2000). Brown *et al*. (1999) identified nearly 80% of primary undergraduate pre-service students with similar reservations about mathematics subject matter knowledge and their experience of learning it, but found that primary trainees initially negative attitudes were significantly ameliorated by mathematics methods courses in the university.

One major finding of Askew *et al*. (1997a) was that serving teachers' belief systems - concerning mathematics, mathematics teaching and children's learning - are associated with teaching effectiveness. Of six case study teachers found to be highly effective, all but one gave evidence of strongly 'connectionist' beliefs. The connectionist orientation is characterised by the belief that most children are able to learn mathematics, given appropriate teaching that explicitly makes links between different aspects of mathematics. Connectionist teachers perceive the semantic unity of mathematics and believe children develop mathematical ideas by being challenged to think through explaining, listening and problem-solving. One of the main research instruments used was a concept mapping interview which probed their understandings of mathematical connections. 'Effective' teachers' response to the interview pointed to good subject knowledge, irrespective of their qualifications, indicating that connectionist 'beliefs' have a hard edge to them.

Conclusion

One striking feature of the UK knowledge base considered in this chapter is its focus on primary pre-service and serving teachers. It would appear that very little is known about the extent or the sufficiency of the subject matter knowledge of secondary mathematics teachers. Yet research in the USA in the 1980s and early 1990s points to the conclusion that the adequacy for teaching of the mathematics acquired as a mathematics undergraduate cannot be taken for granted. Perhaps research into primary teachers' subject matter knowledge, and that of primary student teachers in particular, is received with less resentment and suspicion because most primary generalists would themselves recognise the limitations of their mathematics knowledge for teaching. As we note in Chapter 4 of this monograph, primary teachers have a record of willing engagement with development opportunities in mathematics.

Consistent with the findings of Ball and others in the USA, it would appear that formal qualifications (such as 'A level') in mathematics are not reliable indicators of effective mathematics teaching in primary years. Wilson *et al*. (2001, pp. 5-12) note a "threshold effect" in that study of mathematics "beyond five undergraduate courses" has minimal effect on teaching effectiveness. The culture of UK research is such that the "course" is not taken to represent some quantum of mathematics knowledge, and we know of no comparable UK finding. On the other hand, Wilson *et al*. report that "We reviewed no research *that directly assessed prospective teachers' subject matter knowledge* … To date, researchers … have relied on *proxies* for subject-matter knowledge, such as majors or coursework" (2001, p. 6, our emphasis). In this particular sense, the UK research clearly offers a distinctive contribution in relation to primary pre-service teachers.

Secure knowledge of mathematics – its modes of inquiry and the integrity or 'connectedness' of its content – is clearly associated with primary mathematics teaching judged to be effective or consistent with norms of good practice. By ‘secure’ we mean what Ma (1999) calls ‘profound understanding’, entailing the “breadth, depth, connectedness and thoroughness of a teacher’s conceptual understanding of mathematics” (Ma, 1999, p. 120). Clearly this is a matter of degree rather than simple presence or absence. Yet there is little evidence to suggest that teachers’ mathematics subject matter knowledge develops as a consequence of teaching. The difficulty in addressing primary pre-service teachers’ weak syntactic knowledge in the training year is a cause for considerable concern; indeed, there are no grounds for supposing that the issue is tackled at any later stage.

Research activity in this area – both theoretical and empirical – is still in its infancy, with the work of Shulman still regarded as some intellectual baseline. His work has been successful in refocusing attention onto subject-specific aspects of teaching knowledge where once it had been on generic pedagogic factors. Corbin (2000, p. 2) argues that models of the knowledge required for teaching mathematics “cannot be exhaustive or complete in themselves; part of their usefulness can be at their borders, in what they specifically exclude and include in particular instances of their use”. Research about the *transformation* of subject knowledge, for example, does not include explicit detail of how such subject knowledge is held in an intellectual way by teachers, other than by inference from the ways it is demonstrated by the explanations given or the activities chosen. Ignoring one or more dimensions of knowledge for teaching restricts understanding of the whole picture, yet to work with all the variables makes understanding of that picture virtually impossible and inquiry unmanageable.

In conclusion, we identify three key questions, indicating what we believe to be fruitful directions for research in this area.

First, what kind(s) of final year (say) undergraduate mathematics courses might be developed with prospective mathematics teachers in mind? Secondly, what are the conditions most favourable for teachers to enhance their subject knowledge, and how can these be implemented in teacher education? This might differ for primary and secondary teachers, and for ITT and CPD. Finally, in what ways might teachers’ syntactic knowledge of mathematics in particular be addressed and enhanced within ITT and CPD?