# DEVELOPING MATHEMATICS TEACHING AND TEACHERS

## A Research Monograph

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# PREFACE

This monograph focuses on research and practice in the professional development of mathematics teachers and the associated development of mathematics teaching. The professional context is almost entirely UK based, although insights from research have been drawn from elsewhere in order to inform understanding of current practice. The monograph has been written and compiled by a number of mathematics education researchers who are also practitioners in the field. Most are currently teacher-educators and, in addition, have been mathematics teachers in school or higher education for some period of their professional lives. A full list of contributors is provided at the end of this preface.

The origins of the monograph are worthy of note as they are significant to the way in which it developed. Work began as part of the *FRAME* initiative (*Formulating a Research Agenda in Mathematics Education)* organised by the *Committee of Professors in Mathematics Education* (CoPriME) in collaboration with *The British Society for Research in Learning Mathematics* (BSRLM). This initiative was designed to look critically at research in mathematics education in the UK, in order to inform, and possibly to influence, educators and policy-makers. Early discussion led to the formation of a number and variety of areas of interest, as detailed in Morgan and Jones (2001). These were subsequently reduced to form five major groups: Transitions, Curriculum, Mathematics Teaching and Teachers' Professional Education and Development, Teaching, Learning and Assessment, and Mathematics and Society.

A day conference was held in October 2000 to promote the project; each group worked separately and coordinators fed back to the FRAME committee. Group 3, *Mathematics Teaching and Teachers' Professional Education and Development*, used as starting points for discussion a set of short, informal papers contributed by participants and circulated in advance. Following the day conference these papers, and further electronic communication within the group, led to a draft document being produced by a small subgroup. This draft, posted on the BSRLM website, provided advance reading for a FRAME Teaching Development strand at a subsequent BSRLM Conference (Jaworski, 2001). The day meeting, and further electronic contributions and communications, encouraged critical reflection on the draft document. The subgroup subsequently produced a more comprehensive document, part of which was included as one chapter of a FRAME draft report circulated informally to members of the CoPriME/FRAME working groups.

The subgroup decided that work should continue on the full document. From the various contributions, the considerable extent, diversity and complexity of the field became clear. There was a substantial focus in mathematics teacher education with a concern for the development of mathematics teaching that would enhance the learning experiences of students of mathematics, mainly at primary and secondary school levels. Teacher knowledge and its development seemed to be a key element

across phases, with initial teacher education and the continuing professional development of practising teachers as obvious areas for consideration. Finally, important to a UK focus on teaching development across several decades, was the notion of *inquiry* as it related to the teaching of mathematics in classrooms, and the associated learning of teachers.

The original focus, within the FRAME initiative, subsequent meetings associated with BSRLM, and gathering material from colleagues in the UK meant that the monograph was based very firmly in the UK teacher education scene. A guiding principle was to report on major initiatives in teaching development and teacher education in the UK that had an associated research base. In addition, we included references to relevant research literature from a wider field. This took us into areas that went beyond mathematics education and beyond the UK. We tried to show how issues and practices in *mathematics* education related to education more broadly, and how those in the UK fitted into a more international perspective.

The monograph is written primarily for academics and professionals who are concerned with knowledge in mathematics teaching and its development and thus have an interest in literature in the area. This includes teachers, teacher educators and researchers, but might also include education providers and policy makers.

Contributors to the monograph at its various stages have included: Tamara Bibby, Laurinda Brown, Margaret Brown, Tony Brown, Diana Coben, Maria Goulding, Linda Haggarty, Jeremy Hodgen, Barbara Jaworski, Sue Jennings, Sylvia Johnson, Keith Jones, Olwen McNamara, Pat Perks, Stephanie Prestage, Alison Price, Tim Rowland, Sue Sanders, Margaret Sangster, Ian Stevenson and Julian Williams.

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# 1 PROLOGUE

## Introduction

This monograph is a 'professional' review resulting from a 'practitioner' inquiry conducted by a number of mathematics teacher educators and researchers into the initial and continuing education of mathematics teachers in primary and secondary schools, and the associated development of mathematics teaching. The Council of the British Educational Research Association (BERA) defines a professional review (or 'user' review) as one that "can be seen as normally arising from an academic review in such a way that it: is devised and written by researchers and users working together; brings together the findings of trustworthy research studies on significant educational questions of immediate concern to practitioners and/or policy makers; channels them into a user-friendly and trustworthy document, which aims to inform critically the thinking of practitioner and/or policy makers, and thereby stimulates discussion and worthwhile educational action" (Bassey, 2001).

We believe that such a review of research and practice into the processes of mathematics teaching development and mathematics teacher education is valuable and important. First, because these processes are of significance for teachers themselves; secondly, because ultimately they have an effect on pupils learning mathematics in classrooms.

Mathematics education research in recent decades has developed a view of knowledge growth in mathematics as occurring through interactions of human beings in a social world. The ways in which such knowledge is treated in the literature is dependent on the associated theoretical perspectives. A constructivist tradition, associated with the work of Piaget and dealing with the construction of mathematical knowledge by individuals within social settings, has been promulgated largely during the 1970s and 80s (Piaget, 1950; Skemp, 1971; von Glasersfeld, 1987; Cobb, Wood and Yackel, 1990; Ernest, 1991; Confrey, 1995; Steffe, 2000). The focus on individual learning in much of this work was challenged by researchers and theorists working in a sociocultural tradition related to the work of Vygotsky and his followers and seeing knowledge as derivative of language, culture, social setting and human interaction (Wertsch, 1991; Bauersfeld, 1994; Bruner, 1996; Lerman, 1996). There is currently much debate about whether these perspectives are incommensurable, or whether it is possible to bridge the theoretical gap (e.g., Jaworski and Kleve, 2000; Kieran, Foreman and Sfard, 2001).

Fundamentally we are concerned with *teachers'* knowledge; however, in trying to elucidate this issue, a first consideration is the learner of mathematics who might be a pupil in a classroom or a pre-service teacher at university, or in a variety of further educational settings. There is an expectation that students in such settings will learn mathematics. It is also of fundamental concern *what* mathematics will be learned and

*how*. For example, is it sufficient for a student to learn mathematical skills - the algorithms and rules for certain operations, for example - or are we looking for deeper learning in terms of mathematical concepts, relationships, applications and aesthetics? Paradigms of conceptualising knowledge are important, as is mathematical epistemology, and alternative ways of formulating and perceiving mathematics (e.g., Khun, 1962; Lakatos, 1976; Ernest, 1991). The way mathematics is seen, for example, as a *Platonist* construct, a *formalist* process, or a *fallibilistic* enterprise, is believed to underpin ways in which classrooms are constituted, and mathematical activity composed (e.g., Davis and Hersh, 1980; Ernest, 1991; Sanders, 1994). For many teachers of mathematics, such foundations of knowledge and their relations to pedagogy are largely tacit (e.g., Othman, 1995). What is of importance to the teacher is fostering students' growth of mathematical knowledge within some formal educational environment, which includes institutionalised management and curriculum and assessment practices (e.g., Smith, 2001; Othman, 2002). Part of the teacher's responsibility involves creating opportunities for students to interact with mathematics: to deal with mathematical ideas, address mathematical concepts, learn skills, and develop ability to apply mathematical knowledge, as indicated in the National Curriculum document for mathematics (DfEE, 1999a). The fundamental triad of teacher/student/mathematics represents this situation. A slightly different form of this triad replaces student and teacher with the processes of learn*ing* and teach*ing* (i.e., teaching/learning/mathematics) and focuses on ways in which the participants involved engage in these processes. The focus on processes allows us to consider knowledge development without falling into the trap of talking about some people 'developing' others, for example, teacher-educators developing teachers. This can result in a 'deficit' discourse relating pupils or teachers who *lack* knowledge or expertise and thus need remediation (as indicated, for example, by Brown and McIntyre, 1993 and Dawson, 1999).

These considerations are a preamble to exploring questions about teachers' knowledge, the development of that knowledge and ways in which it can be fostered through teacher education practices in order to develop teaching.

## The Structure and Content of the Monograph

The monograph focuses on many questions and associated issues relating to the development of mathematics teaching and the education of teachers of mathematics. It focuses on mathematics and its learning only where such considerations relate directly to the development of mathematics teaching and teachers. In organising what is an extensive and complex set of materials we decided on four areas that seemed to make sense distinctly, although there is inevitably overlap and links between them. They are:

- Subject and pedagogical content knowledge for teaching mathematics (Chapter 2).
- Initial teacher education for teaching mathematics (Chapter 3).

- Continuing professional development in mathematics teaching (Chapter 4).
- Developing mathematics teaching through communities of inquiry and critical intelligence (Chapter 5).

Where we speak of teachers' *knowledge*, we include both knowledge of mathematics and knowledge of teaching and learning mathematics. Discussion in Chapter 2 deals with complexities in both of these domains as well as their inter-related nature. For the former, mathematical knowledge and its nature are important, as well as research relating to people's conceptions of mathematics. The latter draws on wide considerations of learning and teaching knowledge, both in and beyond mathematics education. This includes the domains of teacher knowledge identified by Shulman (e.g. 1987) that include knowledge of subject and of pedagogy and of 'pedagogic content knowledge' that relates pedagogy to subject. Included here also are teachers' knowledge and beliefs; thinking and decision-making; teachers' practices and their development; all of which contribute to the processes of learning and teaching in classrooms.

Further, teachers of mathematics are required to develop an 'official' pedagogic discourse in line with the requirements of the formal educational system. These requirements, some statutory and some non-statutory, guide and constrain learning and teaching in both classrooms and initial teacher education. Chapter 3 looks at Initial Teacher Education, beginning with a brief documentary analysis of the way in which successive governments have, since the 1980s, increasingly shaped it into the official discourse now referred to as Initial Teacher Training (ITT). We explore as researchers/practitioners, and importantly teacher educators, our understanding of these last two decades and chart what for us have been the most significant shifts in direction. We identify, for example: theoretical models of ITT; the emergence and changing context of partnership; the attitudes and beliefs of pre-service teachers; the increase in prescription and regulation relating to ITT courses and the assessment of pre-service teachers, and the policing of both by the Office for Standards in Education (OFSTED) - a significant feature of the current ITT *milieu*.

All forms of knowledge cited above develop further, or have the potential to do so, as teachers engage in the complexities of teaching. The development of mathematics teaching by practising teachers has been formalised in a variety of ways, and is currently gathered under a general heading of Continuing Professional Development (CPD) and we address this in Chapter 4. The term tends to be applied to teachers rather than teaching, and hence can result in the deficit discourse mentioned above. Teacher educators work with teachers in a variety of modes and situations, aiming to engender more 'effective' practices at all levels. (By 'effective' here we include practices whose outcomes are successful in fostering students' mathematical learning, their enjoyment of mathematics and their ability to use and apply mathematics in a variety of contexts. We recognise the complexity of these ideas and address the notion of 'effectiveness' further below.) Providers of CPD include Higher Education Institutions (HEI), Local Education Authorities (LEA), independent consultants and

other teachers. Activities include courses, workshops, conferences, classroom and research or inquiry-based developmental activity. This latter activity has become the grounding for an increasingly important strand of CPD and often results from partnerships between practitioners and teacher educators/researchers in a variety of modes at a variety of levels.

We develop and elaborate the notion of 'inquiry' in Chapter 5. Research-based teaching development relates to notions of 'communities of inquiry' or 'critical intelligence', where all participants in the educational system are regarded as researchers in a mutual learning enterprise. Notions of inquiry raise important questions for this monograph and indeed for teachers, teacher educators and researchers regarding the *nature* and *forms* of knowledge. Such essential epistemological questions are explored in the Epilogue to this monograph, Chapter 6, which also takes up the key issues emerging from earlier chapters.

**Monograph Review Methodology**

We conclude this Prologue with an account of the methodology that we adopted in producing this monograph. The origins of the monograph (see Preface) as an inquiry by FRAME Group 3, focused upon *Mathematics Teaching and Teachers' Professional Education and Development*, determined that we began with three key research questions:

1. What do we know in this area?
2. What do we still need to know?
3. What are the key research questions?

As our 'practitioner inquiry' developed we added two further questions:

4. What is current practice and how is it situated historically?
5. What are the key issues in theory-practice relationships, and what do they indicate for future practice?

In reviewing research and other theoretically-based literature pertaining to questions 1 and 4 'What do we know?' and 'What is current practice and how is it situated historically?' we trawled data bases including BEI, ERIC and the Social Science Citation Index. We also reviewed the contents pages and hand-searched the content of journals in the areas of teacher education and professional development, with a particular focus on mathematics teacher education. Using the 'mathematics-education' e-mail discussion list maintained by Peter Gates at Nottingham, we solicited information concerning research (including local, possibly unpublished initiatives) on the initial training and continuing development of mathematics teachers. In addition we asked other teacher educators and researchers what issues they perceived to be significant. This consultation process included two 1-day conferences, in London and in Manchester, involving some 60 colleagues, who submitted about 20 short issues-based review/summary papers, most of which have

been assimilated in this monograph. Ongoing electronic communication was established with a wider group of individuals as a result of these day conferences. Finally, we benefited from eight reviews of the whole document, or selected chapters, as part of the BSRLM review process. We included virtually all the suggestions offered in these reviews. This process, as well as identifying some additional material thought useful to include in Chapters 2, 3 and 4, also further informed our thinking regarding Chapter 5.

The protocol that we use for the selection of research and theoretically based literature was that it should:

- have been published within the last twenty years;
- have been published in a peer reviewed journal or academic book;
- relate to, or be thought to be directly applicable to, the UK context.

The protocol that we used to select examples of practice was that they should:

- have taken place within the last twenty years;
- relate to the UK context.

On a small number of cases, we chose to depart from these protocols. For example, certain aspects of Japanese and Chinese teachers' professional lives (described in Chapter 4) are not reflected in the UK scene, yet they provide a model of teaching development that usefully grounds the focus on inquiry in Chapter 5.

We make the distinction in this protocol between research and theoretically based literature. It was not within the scope of this inquiry to consider the soundness or the nature of the methodology of research-based literature and in particular of studies that claimed an empirical research base. We considered this function to have been accomplished, to a degree, in the peer review process. Additionally, as a 'professional' rather than 'academic' review (Bassey, 2001) our remit was such that we set out to identify reports of research and practice, but not to assess them systematically (see also Hammersley, 2001). In relation to theoretical literature, much that we have included was originally grounded in a robust empirical base, such as Shulman's categories of Teacher Knowledge. In the cases where the empirical base was not overt we included particular treatises where we felt that the insights afforded augmented and were valuable to the thesis we were developing.

As a final methodological point, we briefly address the notion of *effectiveness*. At the day conferences, particularly the London conference, the problematic nature of the term 'effective' emerged as a significant issue in discussion. At various times in this monograph, we find ourselves almost unable to avoid using the term in relation to 'effective' learning, 'effective' teaching of mathematics or 'effective' programmes of teaching development. The term 'effective' is typically used in an imprecise or undefined way, yet it can have considerable rhetorical force in its assertions. To suggest that an approach to, for example, teaching or management is 'effective' is

often sufficient to commend it. When we use the term in this document we report it in the form, and with the meaning, indicated by the researchers who employ it. Thus, for example, Askew, Brown, Rhodes, Wiliam and Johnson (1997b) define the 'effectiveness' of teachers of numeracy in terms of average pupil learning gains within a school year. The Teacher Training Agency, on the other hand, defines the 'effectiveness' of pre-service teachers in terms of the standards listed in DfEE (1998a) Circular 4/98 (or 2/02 from September 2002). Sometimes, when the term 'effective' is employed in the literature its specific meaning is not explicitly defined. Given the extreme diversity of research contexts and purposes, definitions of effectiveness related to teaching and learning need to be situated both locally and temporally; the term should be defined explicitly or characterised clearly in order to determine when it is being achieved and to what extent, and to make clear the criteria being associated with it. On occasions in the literature where this is not the case, we do not attempt to suggest or impose fixed or global meanings to the word.

# 2 SUBJECT AND PEDAGOGICAL CONTENT KNOWLEDGE FOR TEACHING MATHEMATICS

*This chapter describes and synthesises philosophical and empirical research on the different kinds of knowledge that might be drawn on in teaching mathematics. The theoretical framework draws on the seminal work of Lee Shulman, and we emphasise here the kinds of knowledge most strongly associated with subject matter per se, in this case about mathematics and how learners might engage with and come to know mathematics. We provide accounts of UK studies into the character and extent of such knowledge, and how it relates to teaching in the classroom. We also include discussion of the role of teachers' attitudes to and beliefs about mathematics.*

## Introduction

In recent years official concern over standards of mathematics in England has been driven by two factors. First, a substantial amount of international pupil performance data has become available (e.g. Reynolds and Farrell, 1996), including data from high profile comparative studies such as TIMSS (e.g. Harris, Keys and Fernandes, 1997), which has focused attention on the relatively poor mathematical attainment of British pupils. Secondly, a burgeoning national audit culture of league tables, targets, and inspection evidence has focused attention upon the variation in standards nationally. During this period much of the educational policy and prescription emanating from government bodies has been premised upon the belief that improved pupil performance will be achieved through improving teachers' subject knowledge. This is evident in the close prescription of subject knowledge in the National Curriculum for ITT (DfEE, 1998a), and the requirement for an audit of that knowledge. Additionally, all student teachers are now required to take, and eventually pass, a national 'numeracy skills' test in order to achieve Qualified Teacher Status. Successive governments, over the last decade, have moved towards an increasing level of prescription from "the what of the curriculum content to the how of teaching methods" (OFSTED, 1995, p. 8 quoted in Brown *et al.*, 1999). This policy has recently resulted in the implementation of the National Numeracy Strategy (DfEE, 1999b) and a cascade training programme for all primary teachers [pupil age 4/5-11] and Key Stage 3 [age 11-14] teachers.

This chapter includes accounts of studies of the mathematics content and pedagogical knowledge of pre-service as well as serving teachers. This material has been included here, rather than in the next chapter, because the major issue here is teacher knowledge rather than the particular circumstances of initial teacher education (the focus of Chapter 3).

## Conceptualising Teacher Knowledge

A recurrent theme in the Prologue to this monograph was the diversity of kinds of knowledge that underpin teachers' preparation for teaching and their subsequent

actions in the classroom. Debate about the role of subject knowledge in teaching has spanned the 20th century. Dewey (1904) held that knowledge of subjects included knowledge of inquiry in the particular domain(s), and therefore knowledge of teaching method as he conceived it. Such a view is consistent with that held in the medieval universities, where no distinction was made between knowledge of content and knowledge of how to teach it. Indeed, the modern form of doctoral examination (defence of the thesis) originates in a medieval *inceptio* which was based on the belief that understanding was *demonstrated* by the act of teaching (Shulman, 1986). The philosopher John Wilson (1975) similarly held that comprehension of the logic of concepts offered guidance on how to teach them. In recent years the importance of subject knowledge has been well documented and the lack of it linked to less competent teaching (Wragg, Bennett and Carré, 1989; Bennet and Turner-Bisset, 1993; Simon and Brown, 1996; DES, 1983; OFSTED, 1994; Rowland, Martyn, Barber and Heal, 2000, 2001) and over-reliance on commercial schemes (Millett and Johnson, 1996).

There is a tension in the literature between those who pay little or no attention to teachers' subject knowledge *per se*, those who consider it in isolation, and those who portray the 'mathematical knowledge' necessary to teach 'effectively' as more complex than simply requiring a grasp of the relevant mathematics subject knowledge. During the early 1980s qualitative research based in classrooms explored the organisation of subject knowledge in teaching, but scant attention was paid to how subject knowledge was integrated into planning and classroom actions. Thus, Leinhardt and Smith (1985, p. 8) wrote: "No one asked how subject matter was transformed from the knowledge of the teacher into the content of instruction". The vital component of the complex teaching nexus is now conjectured to be this transformation of mathematical content, or subject matter knowledge (SMK) into a form appropriate to teach. The term 'pedagogical content knowledge' (PCK) was first employed by Shulman (1986) to depict "the particular form of content knowledge that embodies the aspects of content most germane to its teachability" (*ibid*., p. 9). This pedagogical repackaging of mathematics necessitates facility with the representations, illustrations, examples, explanations and analogies to make mathematical ideas comprehensible to others.

The notion that the development of PCK is the most important element of teacher knowledge to underpin teaching (including elementary school teaching) is currently popular both in Britain and America. Shulman (1986) argued that too much emphasis had been placed in recent (American) research on general pedagogical processes and not enough on subject matter knowledge, attention to which he called "the missing paradigm". In a seminal text, he subsequently delineated seven categories of teacher knowledge:

- content knowledge - both 'substantive' and 'syntactic' (see below);
- general pedagogical knowledge - generic principles of classroom management;
- curriculum knowledge - materials and programmes;

- pedagogical content knowledge - which for a given subject area includes forms of representation of concepts, useful analogies, examples, demonstrations;
- knowledge of learners;
- knowledge of educational contexts, communities and cultures;
- knowledge of educational purposes and values.

(Shulman, 1987, p. 8; see also Wilson, Shulman and Richert, 1987, p. 113)

Bennett and Carré (1993) observed that the relationship between these bases, and the nature of their connection with classroom performance, was by no means clear: they nevertheless affirmed the framework as a "useful starting point in conceptualising students' learning to teach" (p. 7). Even and Tirosh (1995) draw attention to the lack of evidence to support and illustrate the supposed interrelation between SMK and PCK, and suggest that this might, in part, be due to different conceptions of the role of the teacher. At one extreme, the teacher is viewed as the manager of an expert-made curriculum and teacher-proof materials. At the other, s/he is is more engaged with the subject matter itself and ways of representing it in appropriate ways. Even and Tirosh add the telling comment that "research on learning and learners, and research on teaching and teachers have been following separate tracks for a long time" (p. 3). Their research demonstrates the relationship (for a group of Israeli secondary mathematics teachers) between SMK and knowledge of students' common conceptions and ways of thinking on the one hand, and their presentation of subject matter and response to students' ideas and questions on the other.

Critics, however, claim the framework to be not sufficiently dynamic to allow for a non-absolutist view of mathematics (Meredith, 1995), decontextualised (Stones, 1992) and presenting a simple transmission view of teaching (Meredith, 1993; McNamara, 1991; McEwan and Bull, 1991). McNamara (1991) questions whether the distinction between content knowledge and pedagogic knowledge can and should be made, since all mathematics subject matter is itself a form of representation.

These critiques must be evaluated against Shulman's (1987) own clear admission that his framework was provisional, tentative and most probably incomplete. There is, we would suggest, an undeniable sharpness of insight in Shulman's analysis – a blend of philosophy and empirical reasoning – that transcends the critique. A more recent elaboration and refinement of Shulman's scheme, explicitly related to current developments in ITT in the UK, can be found in Turner-Bisset (1999).

The focus of the present chapter is on what Shulman called "the knowledge that grows in the minds of teachers [of mathematics], with special emphasis on content." (1986, p. 9). These correspond to the first, third and fourth of Shulman's seven categories listed above. The plethora of similar but not-quite-identical names for similar concepts is a potential source of confusion here, not least for the author himself. In a footnote, Shulman (1987, p. 8) writes, "I have attempted this list in other publications, though, admittedly, not with great cross-article consistency".

Grossman, Wilson and Shulman (1989), drawing on data from the Stanford University *Knowledge Growth in a Profession* programme, propose three dimensions important to the task of teaching: factual information, central concepts and organising principles of the discipline. Content knowledge (they argue) would appear to emerge through a process of critical analysis, which is guided by the substantive and syntactic structures of a discipline. Broadly speaking, substantive knowledge can be characterised as including knowledge of facts and concepts, and the ways that they are organised. Syntactic knowledge is about the nature of inquiry in the field, and the mechanisms through which new knowledge is introduced and accepted in that community; it includes knowledge about proofs and rules of structures (Schwab, 1978). Such dimensions, implicit in the undergraduate learning of mathematics must, it is conjectured, be rendered explicit in order to teach mathematics. Knowledge of substantive and syntactic structures has implications for what teachers choose to teach, and how they teach (Shulman, 1986, p. 9). The emphasis for Shulman is on key problematic moments of 'contradiction' and 'incompatibility', on transition and process as well as a relational view of knowledge (Corbin, 2000). Shulman also distinguishes between two kinds of understanding: knowing 'that' and knowing 'why': "The teacher need not only understand that something is so; the teacher must further understand why it is so" (1986, p. 9). Skemp (1976) also provided powerful arguments for this position in his classic paper on instrumental and relational knowledge in mathematics.

Feiman-Nemser and Buchmann (1985) introduce the concept of self as they characterise 'pedagogical thinking' in terms of 'strategic, imaginative, and grounded in knowledge of self, children and subject matter'. Askew, Brown, Rhodes, Johnson and Wiliam (1997a) identify as a framework for analysis of effective teacher knowledge and practices: teacher beliefs, pupil responses and teacher pedagogic content knowledge (dependent in turn upon knowledge of teaching approaches, knowledge of pupils and subject knowledge). Indeed, Askew (1999) argues that all propositional statements (beliefs, concepts, knowledge) are constructed through discourse and hence delineation can only be established by social definition. There is evidence (Brown *et al*., 1999) to indicate that such a 'playing down' of the image of 'hard' knowledge is useful and comforting for primary teachers in the process of change. Brown *et al*. (1999) also suggest the need to reconcile the dualities of phenomenological and hard-edged, official versions of mathematics. Such observations implicitly recognise the philosophical differences underpinning different perspectives on mathematics, and their influence on the ways that teachers think about mathematics (e.g. Sanders, 1994).

## Teachers' Knowledge in Action

Much research into the area of subject knowledge and pedagogical content knowledge (e.g. Shulman, 1986; Wilson *et al*. 1987; Tamir 1988; Aubrey 1997) focuses upon the key notion of the *transformation* of subject matter knowledge for the classroom, in relation to teachers' knowledge about explanations, tasks and

activities, and styles of teaching and learning. Shulman reports an episode where, in his estimation, lack of content knowledge was the underlying reason for less effective teaching (1987, pp. 17-18). The Stanford programme shows teachers using various coping strategies when they lack subject matter knowledge, including relying heavily on 'the textbook' (*c.f.* Millett and Johnson, 1996, pp. 54-74) and avoiding discussion and questions. One case study contrasts Joe, who had been a doctoral mathematics student, with Laura, a 'non-math major'; Joe allowed students to generate and evaluate their own methods for solving problems, whereas Laura 'drilled' students in the book-algorithm and was reluctant for them to use any other methods (Steinberg, Haymore and Marks, 1985). Later Shulman and others working in this area (Wilson *et al*., 1987) developed 'intellectual histories' through a series of semi-structured interviews and outlined a six stage model of pedagogical reasoning in action: comprehension, transformation, instruction, evaluation, reflection, new comprehension (Wilson *et al*. 1987, p. 119). They claim that making the transition from novice to expert involves developing many different representations of the same knowledge.

The work of Deborah Ball on the mathematics knowledge of prospective teachers has been influential in the USA and beyond. Ball echoes Shulman's constructs of substantive and syntactic knowledge in any discipline by making a distinction between knowledge *of* mathematics (meanings and underlying procedures) and knowledge *about* mathematics (what makes something true or reasonable in mathematics). Investigating both elementary and secondary pre-service teachers' understanding of division, she found (Ball, 1990a) that both had significant difficulties with the *meaning* of division by fractions. Most could do the calculations, but their explanations were rule-bound, with a reliance on memorising rather than conceptual understanding. They believed that mathematics *could* be meaningful but lacked the knowledge of these meanings themselves. Ball argues that not only should mathematics be revisited in initial training, but also that pre-service teachers may also need to 'unlearn' what they know and believe about the teaching and learning of mathematics.

In an earlier study, Ball (1988) explored the distinction between what mathematics should be known and how it should be organized. The 'what' tends to be check-listed, as in the regulations for ITT (DfEE, 1998a), whereas the 'how' is typically described qualitatively by words such as 'flexibly', or 'in-depth'. Her term 'connected' (Ball, 1990b), derived from comparison of the knowledge held by expert and novice teachers, has since been used by others as a way of describing the quality of subject knowledge that teachers need (e.g. Askew *et al*., 1997a).

In the UK, Aubrey (1993, 1994a, 1994b, 1995, 1996, 1997) observed classroom processes to unfold a tripartite relationship between children's informal mathematical knowledge; teachers' subject and curriculum knowledge, values and beliefs; and classroom practice (exemplifying in her view pedagogic subject knowledge). In an analysis of the mathematical pedagogical subject knowledge of four experienced

'reception' class [age 4-5] teachers, Aubrey illustrates the co-ordination and use of teacher and pupil knowledge in the complexity of ongoing classroom processes (Aubrey, 1997). She concludes that subject content knowledge has a crucial effect on pedagogical practice "even at this early stage of schooling" (p. 144). In the case of one teacher secure knowledge of content and pedagogy gave her confidence to set up explorations, bring out mathematical relationships and display them in various ways; whereas, another teacher lacked a firm grasp of subject matter and was unable to develop explanations or questions effectively. Aubrey emphasises the centrality of subject content knowledge and knowledge of pupil competence to the teacher's pedagogical subject knowledge. This dynamic process of knowledge development requires teachers to have a rich and deep understanding of the main conceptual fields, as well as a grasp of their interconnectedness.

Prestage and Perks (1999a) question the assumption that teachers have full access to subject matter knowledge. They argue that for both experienced and novice teachers much subject matter knowledge remains as 'learner-knowledge' and is not transformed into 'teacher-knowledge' (a case study of 'Frances' in Rowland *et al*., 2000 also describes conditions under which this transformation did and did not – for her – take place). The capacity to transform personal understanding was posited by Prestage and Perks to depend on what teachers brought to the classroom; many experienced teachers used their own mathematical experiences as a pupil in the classroom as a foundation for making decisions (Prestage, 1999). There was little evidence to suggest that teachers' subject matter knowledge developed as a consequence of teaching.

Indeed, Grossman *et al*. (1989) had also argued that teacher educators ought to consider how best to introduce the teaching of subject matter knowledge into initial teacher education, since it could not be assumed that this knowledge would have been acquired through undergraduate study in other departments. Their findings, supported by Aubrey (1997), give yet further evidence that while knowledge of learning, teaching and classrooms increases with experience, knowledge of subject content does not. The situational nature of teachers' decisions and rationalisations about classroom tasks was the focus of the MAST project in which Simon and Brown (1996, 1997) found that "different ways of developing teachers' subject knowledge need to be investigated and evaluated" (1997, p. 7). There is evidence to suggest that teachers need to work in ways that relate subject knowledge and pedagogy at the level they will teach. In the United States, for example, Schifter (2001) has worked with teachers on the subject knowledge relevant to their classroom teaching, using methods directly applicable to ways they might work with students.

## Predictors of Teacher 'Effectiveness'

In a review of USA research dating from the 1970s, Grossman *et al*. (1989) found no statistical significance regarding the relation of subject knowledge to effective teaching (as measured by 'student achievement'). Wilson *et al*. (1987), reviewing research from these early studies, also found that they failed to yield consistent

findings which correlated teacher knowledge with student achievement: thus more teacher subject knowledge did not necessarily mean better teaching. Byrne (1983) suggested that one reason why these earlier studies failed to establish a relationship between teacher subject matter knowledge and student achievement is that both notions had been inadequately conceptualised and measured - the first, typically, by multiple-choice tests and quantity of tuition received, the second by standardised tests. Research by Begle (1968, 1979), and later Ball (1990b), also challenged the assumption that the more subject knowledge teachers had (as measured by quantity of instruction received) the more 'effective' they were. Indeed, they questioned whether some aspects of higher level mathematics training might actually be counterproductive in preparation for teaching.

In a recent study of primary teachers in the UK, Askew *et al*. (1997a, p. 65) also found that 'more' was not necessarily 'better' when they correlated the teachers' mathematical knowledge, measured in terms of qualifications, against pupil learning over one academic year. However, the study found that those teachers who had knowledge and awareness of conceptual connections within the primary mathematics curriculum were likely to be more effective (as measured by average pupil test gains). The study cohort of 90 teachers included three with mathematics degrees and a further 10 who had studied A-level mathematics, of whom 8 had passed. The median class gain was slightly lower for the teachers who had studied A level compared with the other teachers. Thus there was a slight, but not statistically significant, negative correlation between level of mathematical qualification and 'effective' teaching: indicating that mathematical qualifications are not (in isolation) a reliable indicator of the mathematical knowledge required for teaching. One graduate was reported to be unable to explain the links between elementary concepts such as fractions and decimals. In fact, the amount of extended in-service, continuing professional development in mathematics education (such as '20-day' courses: see Chapter 4 for details) undertaken by the teachers was found to be a better predictor of 'effectiveness' than formal qualifications (Askew *et al*., 1997a, pp. 74-79).

International comparisons of pupil performance yield much interesting data upon which to speculate by way of explanation. Chinese pupils, for example, outperform their USA contemporaries. Ma (1999) contrasted the mathematics content knowledge and PCK of USA elementary school teachers with their counterparts in China. She found that the knowledge of the USA teachers studied was relatively instrumental, unconnected and devoid of conceptual grounding. The Chinese teachers, with fewer years of formal education and inferior mathematical qualifications, had acquired a strong conceptual grounding in mathematics (which Ma calls 'profound understanding of fundamental mathematics') that influenced the ways in which they worked with children.

**Pre-Service Teachers' Mathematics Content Knowledge**

Mathematics and English components of primary teacher training courses have been judged to be amongst the most satisfactory when compared with other subjects (DES,

1991b). Nevertheless, the change in subject matter knowledge of mathematics of primary student teachers during training was found to be not significant by Carré and Ernest (1993) in a Leverhulme-funded study of 59 primary Postgraduate Certificate in Education (PGCE) trainees who chose to specialise in one of mathematics, science, music and 'early years' (Bennett and Carré, 1993). Instruments developed for the assessment of knowledge for teaching mathematics – drawing on Shulman's categories - took into account students' beliefs about mathematics, in addition to content knowledge and pedagogical application. Higher order aspects of substantive and syntactic knowledge were assessed, as were students' attitudes (interest and confidence) towards mathematics – a teacher characteristic identified by Ernest (1989) as significant, yet absent from Shulman's analysis. The mathematics and science specialists performed significantly better than the music and early years groups in the pre-tests of content knowledge; the syntactic knowledge of mathematics specialists was dramatically superior to that of the other three groups in an item about the strategies needed to solve a problem. In the post-test, the music specialists showed the greatest gains in content knowledge for teaching, although their improvements were in content and pedagogical application and not in higher level knowledge of structure and inquiry. The study also compared the teaching of mathematics, science and music by the 'specialists', in comparison with the whole cohort. When classroom teaching performance was assessed, it transpired that there was "virtually nothing to distinguish mathematicians and others in teaching mathematics" (p. 161). The situation was very different in the teaching of music, where specialists were judged to perform at a higher level of competence than other students. This was in accordance with the researchers' expectations (p. 164), since music specialist students had been assessed as having a high level of music subject-matter knowledge relative to other students, while this was not the case for mathematics specialists *vis a vis* mathematics.

The relation between subject knowledge and 'effective' teaching was investigated by Rowland, Martyn, Barber and Heal (2000) in a study of the audit and remediation of 154 primary pre-service (PGCE) teachers at one University in the academic year 1997-98. A 16-item test was administered to them some four months into their one-year course, after the main content areas had been 'covered' but before the first of two extended school-based placements. Items on generalisation and proof were found to be among the most demanding. On the basis of this audit, the level of each student's subject knowledge was categorised as low, medium or high, corresponding to the need for significant remedial support, modest support (or self-remediation), or none. On the final school placement, specific assessments of the students' teaching of number were made (against the 'standards' set out in DfEE, 1998a) on a three-point scale weak/capable/strong. There was an association between mathematics subject knowledge (as assessed by the audit) and competence in teaching number. Further analysis identified that students obtaining high (or even middle) scores on the audit are significantly more likely to be assessed as strong numeracy teachers than those with low scores; students with low audit scores are more likely than other students to

be assessed as weak numeracy teachers. In a follow-up study, data for the subsequent (and larger) 1998-99 cohort were subjected to similar analysis, with similar findings. Following Bennett and Turner-Bisset (1993), more extensive data from school placements enabled comparison of mathematics subject knowledge with teaching performance on both first and second placements also with respect to both 'preactive' (related to planning and self-evaluation) and 'interactive' (related to the management of the lesson in progress) aspects of mathematics teaching. The association between audit score and teaching performance was significant ($p<5\%$) in three of the four analyses (Rowland *et al*., 2001), the exception being the first placement/preactive analysis (with $p=8\%$). It should be noted that the mathematics subject knowledge of these students was determined by means of an instrument designed for the purpose, and this instrument (as opposed to formal qualifications gained sometime in the past) provided the three subject matter knowledge categories for comparison with classroom performance. Indeed, there was again no simple relationship between level of formal qualification in mathematics (gained several years earlier) and the audited level of mathematics subject knowledge. The 27 (out of 154) students with an 'A' level pass were distributed throughout the 'top' two-thirds on the audit. An independent statistical analysis (Proctor, 2001) of the 1998-99 data incorporates hitherto unexamined variables in an attempt to 'predict' the mathematics teaching competence of these trainees (see also Rowland, 2001). These additional variables are the gender of the trainee, their chosen age specialism (Early Years 3-8 or Middle Years 7-11), and a subject knowledge confidence self-assessment incorporated into the audit. Proctor observes that although there is no significant gender factor in the audited subject knowledge, the males were more confident (as a group) than the females, the female/Early Years students being the least confident group. As a group, females were more competent teachers of mathematics than the males. Regarding teaching competence, the predictive effect of the self-assessment score is negligible for the Middle Years group compared with that of the audit score and the gender effect, whereas for the Early Years trainees confidence is the more powerful predictor.

At another UK university, Goulding and Suggate (2001) also conducted an analysis of the responses of 201 primary student teachers' to an audit of their subject knowledge. They reported errors on questions in the following domains: appropriate degrees of accuracy in area calculations (84% making errors); proof (61%); calculations involving volume and mass and density (45%); ordering small numbers (44%). They conjectured that problems with measurement may be to do with assessing, estimating and approximating strategies in a written test, although they may also reveal problems with the idea of a continuous number line, and the relationship between mass, volume and density. Ordering small numbers threw up difficulties with ordering when some of the numbers had three or more decimal digits. Most student difficulties seemed to respond to one-to-one remediation, but their problems with proof seem to be of a different nature, being much more difficult to remedy. Goulding and Suggate conjectured that deductive proof was much more

problematic than proof by exhaustion or disproof by contradiction. The proof question also threw up fear of and difficulties with algebra, which were not highlighted to such a degree in other algebra questions on the paper. Rowland *et al*. (2001) also identify aspects of proof as the source of acute difficulty in a cohort of 173 primary trainees, and an apparent reluctance to generate examples when deciding whether a general statement is true or false. The evidence, in the UK, points to the conclusion that deficiencies in syntactic knowledge are especially resistant to remediation in the primary PGCE training year, and systems of support for teachers need to look well beyond pre-service training in this respect.

**Subject Knowledge in Relation to Beliefs and Attitudes**

There is compelling evidence to suggest that experiences as a learner of mathematics, conceptions about the nature of mathematics and instructional practices as a teacher of mathematics are all profoundly interconnected (Thompson, 1984, 1992; Lerman, 1986, 1990; Lampert 1988; Meredith 1993; Sanders, 1994). Beliefs about the nature of mathematics have been found to be "not inconsistent" with dominant pedagogic beliefs (Andrews and Hatch, 1999); and, to play a significant role in shaping teacher behaviours (Askew *et al*., 1997a; Lerman 1986, 1990; Ernest, 1989). Bibby (1999) charts the emotional aspects of primary teachers' mathematical subject knowledge and argues that emotion is a powerful element of change both in positive and negative ways.

A number of studies have investigated the beliefs and attitudes of student teachers. Meredith (1993), for example, in a study of 12 students taken from across three different courses, concluded that students' views about pedagogic content knowledge and their learning of it was not robustly connected to their training. Rather, she concluded, differences in attitudes may have been due to prior learning, knowledge, experience, values and epistemological beliefs. See also Aubrey (e.g. 1997); Carter (1990); Lave and Wenger (1991); McNamara (1991).

Andrews and Hatch (1999) conducted a factor analysis of the conceptions and beliefs about mathematics and its teaching of 577 secondary teachers of mathematics across 200 schools. The study identified five different conceptions of mathematics and five of mathematics teaching. Although dominant conceptions of mathematics manifested in (largely) commensurate beliefs about teaching, the authors found the teachers often possessed disparate conceptions simultaneously and argued that this may be a consequence of "cultural and curricular ambiguities in respect of mathematics teaching in England" (p. 203).

The negative attitude of some secondary mathematics PGCE students towards their experience of learning mathematics as a subject at undergraduate level is evident from some recent work of Anderson, Goulding, Hatch, Love, Morgan, Rodd and Shiu (2000). Brown *et al*. (1999) identified nearly 80% of primary undergraduate pre-service students with similar reservations about mathematics subject matter knowledge and their experience of learning it, but found that primary trainees initially

negative attitudes were significantly ameliorated by mathematics methods courses in the university.

One major finding of Askew *et al*. (1997a) was that serving teachers' belief systems - concerning mathematics, mathematics teaching and children's learning - are associated with teaching effectiveness. Of six case study teachers found to be highly effective, all but one gave evidence of strongly 'connectionist' beliefs. The connectionist orientation is characterised by the belief that most children are able to learn mathematics, given appropriate teaching that explicitly makes links between different aspects of mathematics. Connectionist teachers perceive the semantic unity of mathematics and believe children develop mathematical ideas by being challenged to think through explaining, listening and problem-solving. One of the main research instruments used was a concept mapping interview which probed their understandings of mathematical connections. 'Effective' teachers' response to the interview pointed to good subject knowledge, irrespective of their qualifications, indicating that connectionist 'beliefs' have a hard edge to them.

## Conclusion

One striking feature of the UK knowledge base considered in this chapter is its focus on primary pre-service and serving teachers. It would appear that very little is known about the extent or the sufficiency of the subject matter knowledge of secondary mathematics teachers. Yet research in the USA in the 1980s and early 1990s points to the conclusion that the adequacy for teaching of the mathematics acquired as a mathematics undergraduate cannot be taken for granted. Perhaps research into primary teachers' subject matter knowledge, and that of primary student teachers in particular, is received with less resentment and suspicion because most primary generalists would themselves recognise the limitations of their mathematics knowledge for teaching. As we note in Chapter 4 of this monograph, primary teachers have a record of willing engagement with development opportunities in mathematics.

Consistent with the findings of Ball and others in the USA, it would appear that formal qualifications (such as 'A level') in mathematics are not reliable indicators of effective mathematics teaching in primary years. Wilson *et al*. (2001, pp. 5-12) note a "threshold effect" in that study of mathematics "beyond five undergraduate courses" has minimal effect on teaching effectiveness. The culture of UK research is such that the "course" is not taken to represent some quantum of mathematics knowledge, and we know of no comparable UK finding. On the other hand, Wilson *et al*. report that "We reviewed no research *that directly assessed prospective teachers' subject matter knowledge* … To date, researchers … have relied on *proxies* for subject-matter knowledge, such as majors or coursework" (2001, p. 6, our emphasis). In this particular sense, the UK research clearly offers a distinctive contribution in relation to primary pre-service teachers.

Secure knowledge of mathematics – its modes of inquiry and the integrity or 'connectedness' of its content – is clearly associated with primary mathematics

teaching judged to be effective or consistent with norms of good practice. By 'secure' we mean what Ma (1999) calls 'profound understanding', entailing the "breadth, depth, connectedness and thoroughness of a teacher's conceptual understanding of mathematics" (Ma, 1999, p. 120). Clearly this is a matter of degree rather than simple presence or absence. Yet there is little evidence to suggest that teachers' mathematics subject matter knowledge develops as a consequence of teaching. The difficulty in addressing primary pre-service teachers' weak syntactic knowledge in the training year is a cause for considerable concern; indeed, there are no grounds for supposing that the issue is tackled at any later stage.

Research activity in this area – both theoretical and empirical – is still in its infancy, with the work of Shulman still regarded as some intellectual baseline. His work has been successful in refocusing attention onto subject-specific aspects of teaching knowledge where once it had been on generic pedagogic factors. Corbin (2000, p. 2) argues that models of the knowledge required for teaching mathematics "cannot be exhaustive or complete in themselves; part of their usefulness can be at their borders, in what they specifically exclude and include in particular instances of their use". Research about the *transformation* of subject knowledge, for example, does not include explicit detail of how such subject knowledge is held in an intellectual way by teachers, other than by inference from the ways it is demonstrated by the explanations given or the activities chosen. Ignoring one or more dimensions of knowledge for teaching restricts understanding of the whole picture, yet to work with all the variables makes understanding of that picture virtually impossible and inquiry unmanageable.

In conclusion, we identify three key questions, indicating what we believe to be fruitful directions for research in this area.

First, what kind(s) of final year (say) undergraduate mathematics courses might be developed with prospective mathematics teachers in mind? Secondly, what are the conditions most favourable for teachers to enhance their subject knowledge, and how can these be implemented in teacher education? This might differ for primary and secondary teachers, and for ITT and CPD. Finally, in what ways might teachers' syntactic knowledge of mathematics in particular be addressed and enhanced within ITT and CPD?

# 3 INITIAL TEACHER EDUCATION FOR TEACHING MATHEMATICS

*This chapter examines current practice in the initial education of pre-service primary and secondary mathematics teachers. It focuses on models of training, partnership and the role of the school-based mentor. Current practice in the UK (and in England in particular) can only be properly appreciated when considered in the context of the political and legislative climate of the recent past. Therefore we begin by reviewing from the perspective of the teacher educator, the political moves that have defined the most significant shifts in direction of initial teacher education over the last two decades. In particular, we report on the perceived effects of the increase in prescription and regulation of courses; the policing of this process by OFSTED; and the assessment of pre-service teachers.*

## Introduction

The apparatus of successive governments has increasingly shaped models of Initial Teacher Education in a way that has resulted in the emergence of a policy-driven, institutional process, now officially referred to as Initial Teacher Training (ITT). The current drive towards centralisation and political control was signalled in the early 1980s with the establishment of the Council for Accreditation of Teacher Education (CATE) in DES Circular 3/84 (DES, 1984) which sought to create a more practically based training, to limit the autonomy of training providers, and to shift the balance of power between Higher Education Institutions (HEIs) and schools by introducing partnership arrangements for the delivery and oversight of ITT courses (DES Circular 24/89, 1989a). Early indications of more far-reaching changes were signified by the establishment of alternative routes into teaching through the school-based Licensed and Articled teacher schemes (DES, 1989b) and the introduction of School-Centred Initial Teacher Training (SCITT) (DFE, 1993c).

MOTE (Modes of Teacher Education), the first national survey of training provision, showed that in 1991, 99% of the 45000 student teachers were still trained on programmes offered through HEIs: 31 universities, 36 Colleges of Higher Education and 22 Polytechnics trained 31%, 42% and 27% of pre-service teachers respectively (Barrett, Whitty, Furlong, Galvin and Barton, 1992). Indeed, in the decade since that first survey, the overall percentage of teachers trained through HEIs had not changed significantly (Furlong, Barton, Whiting and Whitty, 2000).

Indications of yet further drastic reform came in 1993 with the announcement of government proposals to establish the Teacher Training Agency (TTA) as a successor to CATE (DFE, 1993b). The role of the TTA was to include: funding; quality control/assurance; accreditation of training routes; teacher recruitment; and improving the quality of teachers and teaching. This remit progressively broadened to include Continuing Professional Development (CPD), Induction and National Professional Standards. It was not until after the Quinquennial Review of the TTA (DfEE, 1999c) that it slowly began to loose some of this portfolio.

Further attempts in the late 1990s to broaden provision by increasing the hitherto disappointing uptake of 'alternative' routes into teaching and extending the variety of 'traditional' training provision were arguably motivated by a desire to increase recruitment by providing prospective entrants a greater variety of access options, but also by determination to break the near-monopoly of the training providers. These included: the repackaging and relaunch of the Articled and Licensed teacher schemes as Graduate and Registered Teacher Programmes; the introduction of the 'Training School' initiative (2000); and the promotion of flexible (2000) and fast track (2001) routes. The situation became progressively more complex with the increasing variation of more 'traditional' provision: 4-year, and shortened 2-year, primary and secondary BEd undergraduate degree courses; 3-year BA/BSc undergraduate degrees (with Qualified Teacher Status); primary, secondary and Key Stage 2/3 [age 7-14] 1-year Post Graduate Certificate of Education (PGCE) courses; and 2-year PGCE part time and conversion courses. (The latter, mainly located in secondary training provision and in subjects such as mathematics, allow students with degrees in related subjects to train as teachers in a shortage subject area.)

The 'effectiveness' of these various routes is, as yet, unproven and indeed in most cases unresearched. Despite the cautionary note from Wilson *et al.* (2001), relating to the limitations of the USA research base on teacher preparation, the USA is still a good way ahead of the UK in this particular respect. Goldhaber and Brewer (2000), for example, report on a survey of 37000 12th graders in mathematics and 2500 in science taught by over 2000 mathematics and 1300 science teachers who held a variety of forms of teacher 'certification'. Their results indicate that pupils taught by 'standard' certification teachers do significantly better than 'private' or 'no' certification teachers, but no better than those taught by 'emergency' certification teachers.

In comparison with the USA, the UK evidence base pales into insignificance, with only a few notable exceptions. Not only is the UK evidence base extremely fragmentary, relating often to single case studies or contexts, but also much of the literature and research into the domain is generic in content. It is possible to identify only a few studies and commentaries that are specifically mathematical in focus. Much of this subject-specific research relates to the mathematics subject knowledge of pre-service teachers, and has been reviewed in Chapter 2. It is also the case that, in the research literature we have identified relating to the initial education of primary teachers, it was uncommon to find aspects particular to mathematics education covered in isolation from issues relating to more general 'professional' studies, other than in the area of subject knowledge. In addition to the lack of research on 'alternative' routes there is also lack of comparative studies, other than inspection evidence, relating to the 'effectiveness' of the great variety of courses in the more 'traditional' routes into teaching, some of which have admittedly only recently been established.

## Prescription and Regulation of ITT Provision

As we have seen, over the last 20 years successive governments have progressively increased the control mechanisms and regulatory prescription relating to ITT, albeit at times with ideologically conflicting motives. The inexorable shift can be traced historically through official reports, surveys and policy directives (e.g. DES, 1983, 1984, 1989a, b; DfE, 1992, 1993a, b, c; DfEE, 1997, 1998a, b; HMI, 1982, 1983, 1987, 1988a, b, 1991a, b; and, OFSTED, 1992, 1993a, b, 1995a, b, 1998). Focused on the development and assessment of academic and professional competences, the utilitarian, practical and skills-based nature of the recent reforms reflects, not just a rethink of the theory/practice ratio, but a redefinition of 'good' practice in both teaching and ITT.

The most significant effect of the increase in prescription in relation to ITT has been the intensification of courses. Repeated demands to develop the partnership model of delivery and extend the school experience component of courses resulted in considerably greater pressure on the delivery of course components during university-based periods. Circular 24/89 required students on undergraduate primary and secondary ITT courses to spend 100 days in school and all other students 75 days; by 1991 this directive was met by all training providers and exceeded by some (Furlong *et al*., 2000). Circulars 9/92 (DFE, 1992) and 14/93 (DFE, 1993a), however, increased the school-based component of PGCE courses still further to 120 days (secondary) and 90 days (primary). Undergraduate 4-year primary and secondary students were required to spend a minimum of 160 days in school.

Holyoake (1993), in a comparative review of European teacher education, noted that the UK was alone in attempting to erode both the length, and the university-based academic rigour of ITT. Others considered the new proposals 'conservative', 'time constrained' and lacking the flexibility, and the intellectual and professional foundation necessary to prepare teachers for the future (Bines, 1994; Dart and Drake, 1993). Carre and Ernest (1993) studying PGCE students as part of the Leverhulme Primary Project, found that the improvement in their knowledge of mathematics was not significant and expressed concern that an increasingly school-based training would cause it to deteriorate further. A second national survey of training provision 'Changing Modes of Professionalism' (1993-1996) revealed that all courses required to comply with the new regulations by September 1996 had done so, but even course leaders committed to a school-based model felt that the "maximum time in school had been reached". Primary course leaders, in particular, seemed challenged by the new regulations and many were putting off compliance until absolutely necessary (Furlong *et al*., 2000, p. 83).

The reduction in the time spent in non-practice-based activities throughout the 1990s was matched by an increase in the level and detail of prescribed learning. Circulars 9/92 and 14/93, for example, contained lists of competences that had to be achieved by pre-service teachers. The combined result of these two parallel modifications of

ITT courses was, not surprisingly, to render courses over-full, and to squeeze out previously key aspects of curricular and professional development.

The second major change for ITT providers in the mid 1990s was with regard to the assessment of the quality of training provision, now the responsibility of the newly established TTA. Training providers had previously been inspected by Her Majesty's Inspectors (HMI), but provision was made in the 1994 Education Act to extend the remit of the Office for Standards in Education (OFSTED), the body which inspected schools, to include inspection of all training providers. The inspections were of significant consequence to providers since 'non-compliance', or perceived failure to meet standards, could be penalised by a reduction in the allocation of training places.

Campbell and Husbands (2000), in a case study of two primary inspections (1996-97 and 1997-98) at Warwick University, contrasted the 'informed conoisseurship' model, formerly deployed by HMI, to the new 'technicist' model adopted by OFSTED. The move from HMI to OFSTED heralded an era of 'surveillance and control' that professed greater transparency of criteria, through the very public Framework for Assessment of Quality and Standards, and had the potential to lead to greater inter-inspector reliability of assessments and greater consistency of judgements across contexts. The criteria statements, of which the 1997-98 version contained about 160, were not, however, justified with reference to an external evidence-base, piloted or evaluated (Gilroy and Wilcox, 1997). Additionally, Campbell and Husbands claim that OFSTED did not expose to examination other documentation and procedures, such as the inspection moderation process and the exemplification of criteria in the training of new inspectors.

Standards of secondary mathematics ITT would appear, however, to have risen considerably since the early 1990s, judging by the official inspection evidence. HMI (1993) reported on a 1991-92 survey in which one third of university secondary PGCE courses were inspected and secondary mathematics was the only subject in which none of the work seen was better than satisfactory. Six years later, in inspections of secondary mathematics PGCE providers (1996-98), almost 75 percent were judged to be good or better. Analysis of the mathematics PGCE inspection reports, however, revealed considerable variation in how reports were structured and, in particular, in how criteria were applied and judgements expressed (Jones and Sinkinson, 2000). Sinkinson and Jones (2001, p. 221) argued that "attention to the transparency of the inspection process and to matters of validity and reliability is crucial if there is to be confidence in the inspection system". In fact, nearly 80% of ITT providers in a survey conducted by Graham and Nabb (1999) expressed a lack of confidence in the validity and reliability of the process.

A third major change in ITT provision was heralded by the publication of the National Curriculum for ITT (DfEE, 1998a), commonly referred to as Circular 4/98. [It should be noted that, from September 2002, the regulations set down in Circular 4/98 will be replaced by those contained in *Qualifying to Teach: Professional Standards for Qualified Teacher Status and Requirement for Initial Teacher Training*

(DfES, 2002) and exemplified in a handbook of non-statutory guidance.] Furlong *et al*., (2000, p. 103) reported in their 1996 survey that course leaders were already finding curriculum planning an "increasingly technical affair" and one in which "more and more issues had to be packed into less and less time". Financial pressures, as a result of the apportioning of student funding between HEI and school in respect of the latter's greatly increased role, were also reported to have increased pressure on the delivery of the courses by increasing the casualisation of the workforce, threatening job security and increasing staff/student ratios.

**The Standards Agenda**

One of the TTA's first major projects was to devise a set of standards for the teaching profession. The National Standards for the Award of Qualified Teacher Status (QTS) (TTA, 1997b) was one of a portfolio of Professional Standards that included those for subject leaders, Special Educational Needs Coordinators (SENCOs) and Headteachers (TTA, 1998). Following in quick succession came the Career Entry Profile (1997) and the National Curriculum for ITT (DfEE Circular 4/98, 1998a) with a focus on subject knowledge and the requirement that providers audit primary pre-service teachers' subject knowledge in Mathematics, English, Science and Information and Communications Technology (ICT). The combined weight of these assessment frameworks functioned as a quality assurance mechanism for newly qualified teachers. Ultimately also, the assessment of students through the OFSTED Framework for Assessment of Quality and Standards was a crucial mechanism for the regulation of ITT providers. The TTA aimed to generate 'consistent and reliable' assessment practices within and across training provision. In a survey of ITT providers, however, Mahony and Hextall (2000) found that the external assessment portfolio of QTS standards, Career Entry Profiles and Circular 4/98, when combined with locally negotiated practice, amounted to a 'bureaucratic nightmare'. Although 63% of tutors felt course planning and assessment would improve as a result of QTS Standards only 8% felt that the overall quality of their courses had improved. It is arguable that rather than generating 'consistent and reliable' assessment practices inconsistent messages were again being conveyed in terms of the way in which different types of knowledge and assessment regimes were prioritised.

This tension was particularly apparent when, in addition to the audit of mathematics subject knowledge, a Numeracy Skills Test for all prospective teachers (primary and secondary) was introduced. The piloting of the 'paper and pencil' QTS Numeracy Skills Test, in 2000 was closely followed by the planned introduction of the computerised Numeracy, Literacy and ICT Tests in 2001 (the latter was abandoned in 2001 as a result of software problems). Two studies, Hextall, Mahony and Menter (2001) and McNamara, Roberts, Basit and Brown (2002), conducted in June 2000, reported pre-service teachers feeling extreme anxiety. They felt 'insulted' by the Numeracy Skills Test and a 'deep sense of grievance' that it had devalued existing mathematical qualifications, 'shifted the goal posts' to make them 'jump through more hoops', and, been designed to depict the government as a 'gatekeeper of

standards'. Additionally, the test was not seen to relate to the broad range of mathematical knowledge they required in the classroom, or show understanding of their 'broader professional role'. The technical 'glitches' and organisational expertise involved in the later computerised tests (Hextall *et al*., 2001) made them no less an administrative nightmare than the paper and pencil pilot.

Moreover, at a time when 'inclusion' and recruitment and retention of ethnic minority teachers is high on the government agenda, there is evidence to suggest that certain sections of the cohort may have been disadvantaged (Hextall *et al*., 2001; TTA, 2000). The Numeracy Skills Test may also have other unintended consequences, in that it may cause students' conceptions of mathematics to become more 'hard-edged' and absolutist in conception. ITT has previously been reported to be successful in increasing students' confidence in their ability to teach mathematics and shift such absolutist beliefs (Bennett *et al*., 1993; Carre and Ernest, 1993; Carter, Carré and Bennett, 1993; Brown *et al*., 1999). Additionally, studies (Brown *et al*., 1999; Green and Ollerton, 1999) have identified anxiety about mathematics as a major issue for pre-service primary teachers and the introduction of the 'test' may prove to jeopardise the success of some training courses in remediating the problem.

Over the last decade, however, the improvement reported by OFSTED in the standards of training providers would appear to be reflected in a rise in the standards of Newly Qualified Teachers (NQTs), according again to inspection evidence. In a 1992 survey (OFSTED, 1993) of 32 mathematics lessons taught by NQTs - 20% were graded 'unsatisfactory', 80% 'satisfactory' or 'good' and none 'very good' (quoted in Jones and Sinkinson, 2000). During the years 1997 to 2001 the percentage of 'unsatisfactory' or 'poor' NQTs in secondary schools has reduced by a third and the percentage of 'good' or 'very good' increased by 15%. Over the same period the percentage of 'unsatisfactory' or 'poor' NQTs teaching in primary schools has reduced by a half and the percentage of 'good' or 'very good' increased by around 20%. Recent figures appear to indicate that this upward trend has continued. Now, for the first time ever, over half of all NQTs observed by OFSTED inspectors during their induction year were rated as 'good' or 'very good' and only 5% of NQTs teaching in primary schools, and 8% in secondary schools, were rated by inspectors as 'unsatisfactory' or 'poor' (HMI, 2002, p. 92 quoted in Howson, 2002, p. 32).

## Models of ITT

### *Theoretical conceptualisations*

Aside from the official government model of ITT as a set of skills and competencies outlined in the sections above, a number of other theoretical conceptualisations have been suggested in the literature. These have included: 'apprenticeship of observation' (Lortie, 1975); development of expertise (Berliner, 1988); 'rite of passage' (White, 1989); 'legitimate peripheral participation' (Lave and Wenger, 1991); 'induction of novices by experts into culturally based understandings and skills' (Edwards, 1995); and 'cultural performance' (McNamara *et al*., 2002). A

number of these models have been grounded in a social anthropological frame. Lave and Wenger's (1991) analytical perspective on induction, 'legitimate peripheral participation', for example, sees learning resulting from asymmetric co-participation within a community of practice where development of expertise and understanding is situationally and contextually grounded. Learning in this model is not 'from talk' but 'to talk'; the master/apprentice relation, where it exists, functions to confer legitimacy rather than to provide teaching. Whilst this perspective has gained currrency in education, and mathematics education in particular, critics point out that learning in a school context "requires an understanding of the structure of pedagogy" (Adler, 1996, p. 9) and needs to take into account teaching, as well as learning, in the school as a 'community of practice'. One critique of the application of the 'apprenticeship' model to ITT claims that it reflects a false dichotomy between university and school-based elements, negates the importance of reflection and studies in educational theory, and does not correspond to the views of mentors (Jones, Reid and Bevins, 1997). Such observations are rooted in theoretical perspectives of the nature and growth of knowledge, addressed more explicitly in Chapter 5.

White (1989, p. 177) adopted the classic anthropological three-stage 'rite of passage' model to depict pre-service teachers' education. 'Separation' sees students 'cut the ties that bind them to the ordinary world' of college to embark upon the teaching semester where during their 'transition' into the 'real world' of the classroom they are inducted into the specialised body of knowledge and skills before finally being 'reincorporated' into college life, transformed by their experience. Exploring the applicability of this schema as a learning theory in relation to teacher education Eisenhart, Behm and Romagnano (1991) found it an inappropriate model, in certain ways, for the mathematics programme at one college. In particular, rather than creating experiences that were consistent and persuasive, a coherent ritual transmitted through a 'specialised body of knowledge', it was found that the programme endorsed inconsistent goals and created confused messages.

McNamara *et al*. (2002) present the experience of ITT as a complex matrix involving the student, the HEI, schools and government, in which transition is not linear but involves a back-and-forthness which repeatedly unsettles and repositions the 'initiand' in response to a complex, and often contradictory, set of agendas. Students depict their experience of embryonic teacherhood as 'game', 'play' and 'performance', interspersed by a series of 'ordeals' that legitimated their passage into QTS.

A picture of disparate, if not overtly conflicting, discourses and agendas, together with incompatible assessment practices, are reflected in some of these theoretical conceptualisations of ITT. This is indicative of fragmentation of, and lack of clarity about, the principles, practices and philosophies underpinning ITT; such outcomes are characteristically a potential hazard of current government-led HEI/school partnership model of ITT. Studies indicate that presenting pre-service teachers with

competing demands could (McNally, Cope, Inglis and Stronach, 1994, p. 229) "create a role conflict in which they fail to achieve a sense of belonging and confirmation of teacher status".

Brown, *et al.* (1999) in a study of primary BEd pre-service teachers depicted the training process predicated on supposing that such seemingly divergent demands could be reconciled and offered a theoretical framework for mathematics education comprising three dualities: phenomenological versus official accounts of mathematics; discovery versus transmission conceptions of teaching; and perceptual versus structural accounts of the training process. These pairs of potentially dichotomous perspectives might also be seen as complementary hermeneutic arcs (Ricoeur, 1981). The first of each pair reveals how pre-service teachers' perceptions frame the space they are working in; the second shows how contextual parameters shape their perceptions. Analysis of data in the study, however, pointed to the pre-service teachers pursuing partial accounts of their transition with little attempt at overall reconciliation.

Empirical evidence relating to models of ITT is to be found in the 1991 MOTE survey of training providers, in which over 80% (218) of courses claimed to espouse a particular philosophy or model of professionalism. In over 70% of cases (across primary, secondary, undergraduate and postgraduate) this model was of the 'reflective practitioner'. Only 6%, or 13 courses (mainly primary undergraduate), laid claim to a 'competency model'. Other models reported in the findings were 'theory into practice', 'teacher as researcher', 'problem solving', 'practical', 'teacher as manager', 'active learning', 'child-centred' and 'eclectic' (Barrett *et al.*, 1992).

*The reflective practitioner*

Discourses of reflection (Adler, 1991) in the literature include content free 'reflective teaching' (Cruikshank, 1987), reflection on action leading to 'reflection in action' (Schön, 1983, 1987), and 'reflection as critical inquiry' (Zeichner and Liston, 1987). Reflective processes are often represented in three-level hierarchies (Van Manen, 1977; Carr and Kemmis, 1986; McIntyre, 1993), loosely derived from Habermas (1973), or in protocols such as "reaction, repair, review, research, retheorizing and reformulating" (Griffiths and Tann, 1992). Strategies claimed to foster reflection in pre-service teachers include action research (Carr and Kemmis, 1986; Liston and Zeichner, 1990), case studies (Ross, 1989), micro teaching (Sparks-Langer and Colton, 1991), theorising on the practice of others (McIntyre, 1993) and structured curriculum tasks (Smith, 1991). A number of educators also feel that reflective work provides a forum in which students can reconstruct their own identity as they become inducted into professional discourses (e.g. Hanley and Brown, 1996, 1999; Tann, 1993).

Despite its popularity amongst providers there have been various critiques of the use of 'reflective practitioner' techniques in ITT. Many studies indicate that that most students' reflective thinking and writing remains seriously under-developed, and is

mostly of means/end descriptive character (level one), although Hatton and Smith (1995) found dialogic (level two) reflection occurred in one third of the essays of secondary BEd students based on 'critical friend' peer interviews. McNamara (1990) locates the real difficulty that pre-service teachers face as theorising classroom practice. The latter, it is thought by some, remains a much more productive exercise for experienced practitioners (McIntyre, 1993; McNally *et al*., 1994). Edwards (1995, p. 600) identifies scant evidence in the literature to suggest that "reflection on practice in ITT is an opportunity to connect any sort of pedagogical theory with practice". Tickle (1994) in a study of 150 newly inducted teachers found that only 5% mentioned reflective practice as an important mechanism of support. Students often believed it a distraction from the serious business of learning teach (Zeichner and Liston, 1987), another 'hurdle' to get over (Bolin, 1990). Indeed, some feel that the skills required for reflection are very different from those required in teaching (McNamara, 1990). Outright health warnings for the use of reflection in ITT include potential damage caused by its capacity to expose personal deficiencies (McNamara, 1990), pose threats to classroom teachers (McLaughlin, 1994), produce anxiety, low self-esteem and disempowering self-doubt by emphasising the disparity between actual and intended classroom practice (Leat, 1995).

## Partnership in ITT

A fundamental change during the 1980s and early 90s was the considerable strides made by HEI and schools towards voluntarily developing 'partnerships'. Yet according to Brown *et al.* (1993), in a review of partnership and mentoring literature, 'partnership' is a problematic concept. Crozier, Menter and Pollard (1990), in a study of ITT partnerships, characterise it as a 'slippery and imprecise word'. Government, however, was soon to legislate on the nature of ITT partnerships (Circulars 9/92 and 16/93, DfE, 1992 and DfE, 1993a) and transfer, for example, responsibilities for practical training to schools. Edwards (1995) challenged the government's simplistic depiction (DfE, 1993a) of the student developing practical skills in the school and subject knowledge in the university. Furlong *et al.* (1996, p. 39) claim that HEIs subsequently went through a period when they "lost sight of" and had to redefine their "distinctive contribution" to ITT. Despite the now familiar 'rhetoric of partnership', Taylor (2000, p. 55) believes there is little acknowledgement in the literature of the additional 'cost'; or the relative distribution of 'resources', 'control', 'quality assurance', 'accountability' and 'penalties'. He nevertheless speculates that partnership offers the 'best hope for success': partnership, not a 'quick fix', is a "process not a outcome" in which the "specification of who does what is less important than the existence of shared values based as far as possible on a common knowledge base".

The 'Changing Modes of Professionalism' survey (1993-1996) identified a continuum of ITT-focused relationships that extended from HEI-led pre-1992 ITT courses to school-led SCITTS; although neither of the two extreme models could, they deemed, be considered partnerships (Furlong *et al.,* 2000).

SCITTs were the flagship of the government's plan to extend the variety of training provision, and in particular non-HEI provision. Six consortia of schools (three primary, three secondary) were established in the first instance, taking responsibility for course design, leadership and administration with little or no support from HEIs or other outside consultants. The primary SCITTs did, however, call upon LEA expertise to teach subject application (Furlong *et al*., 2000). The inspection report (OFSTED, 1995a) of their first year of operation declared only one of the consortia "good" from the start and rated two "unsatisfactory". In a survey of primary headteachers, Hannan (1995) found little evidence of support for government proposals (DFE, 1993) to establish more independent school-based primary ITT (including the now infamous 'Mum's Army' scheme – a one year non-graduate scheme for early years teachers), although there was a desire to restructure the present arrangements to give primary schools a more significant role.

'Ideal typical' models of partnership were identified by Furlong *et al*. (2000, p. 78) as 'complementary' and 'collaborative'. The former, characterised by separate roles and responsibilities, was found to be most prevalent and was commonly seen to be a "pragmatic response to limited resources". In the latter model the partners were seen to have different, but equally legitimate, bodies of knowledge. The Oxford Internship Scheme (Benton, 1990), in its time a ground-breaking initiative, was posited as a classic example of this type of partnership. Other examples in the literature have been the Sussex Secondary PGCE partnership (Lacey, 1977; Furlong *et al.*, 1988) and, from a particular curriculum area, the Exeter history PGCE course (Nichol, 1993).

The Oxford Internship Scheme was a one-year secondary PGCE programme, launched in 1987, and developed on the premise of a very close school-HEI partnership that in many ways anticipated the legislation to follow. At any one time about 15 schools were involved with about 10 'interns' attached to each throughout two-thirds of the training year. The course had two components: a curriculum programme and a general programme. The former operated entirely on subject specialism and was the joint responsibility of a school-based subject mentor and a university-based curriculum tutor. Additionally, each school identified a professional tutor who, together with the university-based general tutor attached to the school, was jointly responsible for managing the general programme. McIntyre (1990a) identified six principles underpinning the scheme: (1) partnership – including joint planning of programme and roles between HEI and schools; (2) integration – involving a coherent programme, closely interconnected, allowing theory and practice to be tested against each other; (3) secure learning environments – carefully graduated to ensure students were not anxious or overwhelmed, given the centrality of classroom management issues in pedagogic decision-making; (4) diversity of sources – encouraged to inform student's thinking and teaching; (5) consensus not expected – between HEI and school about any aspects of good practice, divergence should be explored not ignored; (6) emphasis on testing – against a number of criteria, including theoretical coherence, consistency with research evidence and espoused

values, feasibility in relation to resources, expertise, effectiveness and acceptability (See also McIntyre, 1990b).

The 'Changing Modes of Professionalism' survey examined providers' perceptions of the 'location' of the principal course elements: subject studies (undergraduate); main subject application; core and other curriculum (primary) educational/professional studies and school experience. They concluded that, "despite the growing amount of time being spent in schools, there had not been any dramatic shift in the forms of training being undertaken there as opposed to what was offered within higher education" (Furlong *et al*., 2000, p. 83). For example, elements seen by providers to still take place 'mainly' or 'wholly' in HEI were: subject application (by 30% of providers); educational/professional studies (by 40% of providers); and subject studies (by 85% of providers).

**The Role of the Mentor**

A key feature of the new partnership arrangements has been the reconceptualisation of the role of teachers working with students from 'supervisor' to 'mentor'. In the first half of the 1990s the number of teachers formally designated mentors increased more than five-fold (Furlong *et al.,* 2000). Yet questioning the substantiveness of the evidence base about mentoring in ITT, Brown *et al.* (1993, p. 4) identify "little convincing evaluation of effectiveness as opposed to rhetorics of success".

A growing body of literature is now examining this dimension of school-based teacher education (e.g. McIntyre, Hagger and Wilkin, 1993) and positing theoretical models of 'good mentors' (e.g. Jones *et al.*, 1997) and 'influence' (e.g. Jones, 2001). Much of the ITT mentoring literature addresses professional and affective dimensions of the experience, however, and neglects the student as a subject specialist. *Mentoring in Mathematics Teaching* (Jaworski and Watson, 1994) is an exception in this respect as regards mathematics, with chapters that consider the special nature of mathematics (Sanders), the planning and preparation of mathematics teaching (Perks and Prestage) and the mathematics curriculum (French), amongst others.

The Mentor Development Project (Dart and Drake, 1993, p. 177) studied the induction of new secondary ITT mentors by experienced mentors finding that "Mathematics mentors talked a little of investigative mathematics teaching and its difficulties for trainees, but otherwise neither English nor mathematics mentors focused upon subject specific concerns". Brown *et al*. (1999) in a study of 20 primary undergraduate pre-service teachers also found little evidence that either class-teacher/mentor or university (school experience) tutor addressed issues relating to mathematics subject teaching when observing mathematics, focusing instead on classroom management and professional issues.

Smith (2001) focused on the influence of a mathematics teacher, who might also be a school-based mentor, on the improvement of students' teaching of secondary mathematics. The study reported on the nature of written guidance provided by mathematics teachers to a cohort of pre-service teachers of mathematics and

identified the influence of such guidance on them. The findings suggested that teachers advised pre-service teachers most frequently about classroom management, and other aspects of craft knowledge were shared within a framework of class management. Aspects of guidance were generally narrowly focused on traditional mathematics teaching craft skills of explanation, examples and exercises, with some attempt to exhort pre-service teachers to use a wide range of pupil activities.

Perhaps the most detailed account of mathematics mentor activity comes from Haggarty (1995) who, in a rigorously planned and trialled research and development project, studied the implementation of a one-year secondary mathematics PGCE programme at Reading University, modelled upon the Oxford Internship Scheme. The activities of six mathematics mentors in partnership schools were studied as they implemented a mentoring framework, which had been agreed in consultation with the schools. It involved arranging: observation of relevant phenomena and follow up discussions; participation in relevant activities; a teaching programme; discussion of planning; help with self evaluation; input of general perspectives on mathematics topics; and discussion of university tutors' comments. The mentoring process was felt to be the least successful aspect of the implementation of the PGCE programme as a whole. There was evidence that different mentors interpreted their various roles in different ways. One particular cause for concern, however, was that the mentors failed to capitalise on their principal strength, "their unique contextualized understanding of practice" (Haggarty, 1995, p. 104) through which the complexity of classroom reality could be explored. Instead, mentors appeared to model aspects of the tutor's role such as a focus on decontextualised theory. Observation of 'routine' or 'difficult' lessons was not encouraged; students were not observed as often as had been anticipated and often received no significant debriefing discussion. Haggarty surmises that mentors may have experienced difficulty articulating their own practice and that school culture may have precluded them from admitting and discussing their difficulties. A number of key factors were identified as significant in the effectiveness of mentors: views about teacher education, views about mathematics teaching, other responsibilities in the school, relationship with other teachers, and personality.

**Beliefs and Attitudes of Pre-service Teachers**

Haggarty (1995) explored how Lacey's (1977) typology of pre-service teachers' strategies facing problems and displacing blame informed, and was informed by, her study. The strongly held opinions of a number of mentors with regard to mathematics teaching were, for example, identified as potentially counterproductive and contrasted with Lacey's analysis of pre-service teachers' behaviours as 'strategic redefinition', 'strategic compliance' or 'internalised adjustment'. 'Strategic compliance', indicated pre-service teachers' behaviour changes had been brought about under pressure whilst underlying beliefs remained unchanged; whereas 'internalised adjustment', indicated that changes had occurred in both pre-service teachers' behaviour and their beliefs.

Ernest (1989, p. 30) in discussing his model of mathematics teacher's knowledge, beliefs and attitudes identifies components as: 'beliefs' about the conception of the nature of mathematics, models of teaching and learning mathematics, together with principles of education; and 'attitudes' to mathematics and its teaching. He believes that "the crucial factor in developing beliefs and attitudes in teacher education is the form, rather than the content of the learning experiences". Miller and Baker (2001) explore a critical epistemology in which pre-service teachers' models of knowledge and beliefs about teaching and learning were explicitly challenged, and a language developed for describing, understanding and developing practices and positions. They developed, as an analytical tool, a two-dimensional matrix which charted primary student teachers' practices with respect to subject knowledge, beliefs and values, pedagogical positions and power relations, against their moral/social positioning as 'compliant'; 'reflective' and 'interrogative'.

Circular 4/98 specifies the content of training courses, and in particular the subject knowledge component, but it neglects to address the significance of pre-service teachers' beliefs about, and attitudes towards, the nature of mathematics and the processes involved in its teaching and learning. Yet there is a growing corpus of evidence to suggest that beliefs and attitudes can affect the way and extent to which curriculum reform is realised (Cooney, 1988). Andrews and Hatch (1999, p. 220), for example, in a survey of secondary teachers' conceptions of mathematics found evidence that "substantial numbers of serving teachers and many applicants for teacher training, may have perspectives on mathematics which counter the successful fulfilment of [current government] curricular expectations".

As noted earlier in Chapter 2, there is compelling evidence to suggest that experiences as a learner of mathematics, conceptions about the nature of mathematics and instructional practices as a teacher of mathematics are all profoundly interconnected (Thompson, 1984, 1992; Lerman, 1986, 1990; Meredith, 1993). The influences of 'pre-program beliefs' and 'culturally based filters' (Hollingsworth, 1988), employed as interpretive frameworks to make sense of classroom contexts, need to be made explicit and challenged if changes in behaviour are to be achieved (Ahmed, 1987; Bird, Anderson, Sullivan, and Swidler, 1993). The initial transition from school learner to school teacher, if it is to be successful, must often involve a considerable degree of 'unlearning' i.e. discarding of mathematical 'baggage', both in terms of subject misconceptions and attitude problems (Ball, 1988, 1990a, b). Lack of attention to this potential impediment is thought to "help to account for why teacher education is often such a weak intervention - why teachers, in spite of courses and workshops, are most likely to teach math just as they were taught" (Ball, 1988, p. 40).

Opinion is, however, divided as to how much ITT is able to substantively influence beliefs and attitudes. Some research indicates that frameworks are already fairly firmly fixed before training (Tabachnick and Zeichner, 1986; Zeichner, Tabachnick and Densmore, 1987). There is evidence to suggest, for example, that primary pre-service BEd students hold images of teaching formed from their own school days that

are highly influential in moulding their classroom practice (Calderhead and Robson, 1991The beliefs and conceptions of mature entrants to teacher education programmes in mathematics have been found to be dependent on previous career experiences and to be resistant to change (Harel, 1994). Su (1992), in a study of 29 training establishments, found the teaching experience and the class teachers of greatest influence and the course curriculum less so. School pupils were ranked as most significant in helping pre-service teachers to judge their effectiveness as teachers and prior socialisation from former 'inspirational' schoolteachers, family and friends were important in helping them make the decision to enter teaching and remain in the profession. In a study of pre-service elementary teachers' beliefs, conceptions, and practices and their views of mathematical and pedagogical content knowledge, Foss and Kleinsasser (1996) found that their conceptions of mathematics remained constant during a mathematics methods course.

Bramald, Hardman and Leat (1995), however, argue that the effect of training courses do vary and belief systems are not as resistant to change as some research suggests. PGCE courses, for example, have been shown to have significant impact upon students in shifting beliefs about mathematics from absolutism towards relativism (Bennett *et al*., 1993; Carré and Ernest, 1993; Carter *et al.*, 1993). Brown *et al*. (1999) found pre-service teachers experiences, beliefs and attitudes as pupils at school often informed, in a negative sense, their ideal model of teaching in that they adopted 'child-centred' models of teaching. This was, in a number of cases, later modified by the intrusion of pragmatic classroom concerns, so that 'transmission' teaching, so often reviled by them as learner of mathematics at school, became their chosen model of delivery. Cheng (1990) also found humanistic views of teaching and learning, developed in college sessions, tempered by realism after teaching practice experiences.

## Conclusion

As we have seen, Initial Teacher Education has been framed by a number of, at times contrary, structural pressures over the last two decades. First, the proliferation of 'alternative' training routes, added to the increasing variety of more 'traditional' provision and providers, has caused fragmentation and lack of clarity about the principles, practices and philosophies underpinning ITT as a whole. We have little or no understanding of the philosophies that underpin the new 'flexible' routes or 'training schools', or the models of professionalism that are inherent in them. We have little idea of the particular costs and benefits that they pose for mathematics education or mathematics educators; or indeed, of their differential 'effectiveness' in the preparation of groups of pre-service teachers with diverse needs and expectations. Much of the comparative data relating to training provision is from sources such as OFSTED; little independent work of any magnitude has been conducted since the ESRC funded 'Modes of Teacher Education' (1991-1992) and 'Changing Modes of Professionalism' (1993-1996) projects. In-depth reports of practice are typically case studies of particular contexts or comparative studies across two contexts.

The second structural change has been an increased tightening of prescription for the regulation and assessment of training provision. The plethora of government directives relating to all aspects of ITT may have resulted in a stronger collective sense of the task, but only at the expense of diminishing notions of professional autonomy. This has resulted in some university tutors feeling they need to defend their own professionalism, defined in terms of broader educational aspirations, against a framework that to some seems rather narrow. Student teachers have also come under pressure to develop their practice according to government-led agendas, which can result in students specifying their own needs in somewhat restrictive terms, in line with government requirements. Can student teachers develop a capacity for working on their own professional development in a way that relates to their personal aspirations of what it is to be a teacher? How can we as educators model such activity as we balance the competing demands of individual professional autonomy and collective state and student entitlement?

Finally, with regard to partnership, there has been an ideological and practical shift in the context of ITT and the dynamics of the power balance between government, HEI, school and student. There is now a substantial literature on partnership in ITT but, again, much is based on the experience of one provider rather than coming from wide ranging studies. We are as yet unable to answer questions such as those posed by Wilson *et al*. (2001) on the content of pedagogical preparation, instructional methods and contexts best suited for particular aspects of teacher preparation; on the relative contributions of college sessions, assignments, teaching experience, to pre-service teachers' progress in learning to teach; or on the importance of their particular school experience context on the outcome of their practice. Indeed, there seems to be little consensus on even whether consistency across contexts is important. Whilst many studies identify the hazards of conflicting practices and philosophies between HEI and school-based school course components, the Oxford Internship scheme did not expect consensus, and divergence was explicitly 'explored not ignored'. Finally, there has been a plethora of literature on mentoring, a crucial component of partnership models, but we are still unclear about how to develop 'effective' subject mentoring in secondary partnerships. The situation in primary partnerships with regard to subject mentoring is even more challenging and less well-documented.

# 4 CONTINUING PROFESSIONAL DEVELOPMENT IN MATHEMATICS TEACHING

*This chapter considers the development of mathematics teaching through teachers' ongoing in-service professional development. The focus of the chapter is on past and present events and developments within the UK, some reference to research and practice elsewhere establishes a link with subsequent chapters. There is a great deal to be told and to be known about a plethora of UK mathematics in-service initiatives over several decades, yet remarkably little systematic study, evaluation and reflection - what might legitimately be called research - to report. The character of this chapter is therefore principally (but not entirely) historical and descriptive. Where possible, we draw out here what has been learned from these initiatives. Some ways of reflecting on these lessons are introduced in subsequent chapters.*

## Introduction

The need to update and refine professional knowledge throughout their careers has been particularly acute for mathematics teachers since the 1960s, in response to curricular reform, new perspectives on learning and teaching, new technologies and innovations in assessment in all phases of education. Agents foremost in the provision of In-service Education and Training (INSET) have been HEIs, LEAs and Professional Associations, notably the ATM and MA. Mathematics teaching development has also been an intentional dimension of major curriculum development projects such as the School Mathematics Project (SMP), Nuffield Primary Mathematics, Primary Initiatives in Mathematics Education (PrIME), the Raising Achievement in Mathematics Project (RAMP), and the National Numeracy Project 1996-99, which preceded the introduction of the National Numeracy Strategy (NNS). Central government together with various charitable trusts and publishers have provided the funding for these and other initiatives.

1982 saw the publication of the Cockcroft Report (DES, 1982), a landmark report of a three-year government inquiry into the teaching of mathematics in schools. On outcome of the report was the appointment of significant numbers of additional advisory mathematics teachers – so-called 'Cockcroft missionaries' – to LEAs. Nolder and Tytherleigh (1990) give a rare research-focused account of a collaborative curriculum development and INSET initiative between LEA advisory staff and a group of 15 committed secondary teachers. The privatisation characteristic of government education policy since the 1980s (Harland, Kinder, and Keys, 1993) has encouraged significant numbers of independent consultants and agencies to enter the field. From a peak in the mid-late 1980s, the role of HEIs in INSET has declined significantly. A number of former LEA advisory staff have joined the ranks of the independent providers, and many schools have been attracted to a model of self-help, 'school centred' INSET. In a wide-ranging, government-commissioned review of evidence of teaching methods and classroom practices, Alexander, Rose and

Woodhead (1992) stressed the importance of a "mixed economy" INSET, drawing on professional resources within and beyond the school. Their report cautioned that "school-based INSET pursued as the only in-service strategy … may result in some schools merely recycling their own inadequacies" (p. 53).

Since the mid-1990s, the term 'INSET' has given way to 'Continuing Professional Development' (CPD) in official discourse. The fragmented nature of the evidence base regarding the effectiveness of CPD programmes is acute. Furthermore, it is unclear what, if anything, is distinctive about the development of *mathematics* teaching in particular (as opposed to the teaching of any other subject). Nor is there clarity concerning the particular CPD needs of secondary mathematics teachers, as against primary teachers. Few UK in-service education providers seem to have been concerned to research their CPD provision. The more accessible reports of curriculum development projects such as PrIME (Shuard, Walsh, Goodwin, and Worcester, 1991) tend to be strong on the rhetoric of success and weak on objective, systematic evaluation of their impact on teachers or pupils. In any case, there are few studies relating specifically to mathematics and the effects of mathematics training programmes. Indeed research on the effects of INSET programmes, *per se*, has been described by Halpin, Croll and Redman. (1990, p. 164) as "meagre" and lacking a "cumulative dimension".

## CPD and Change

While one purpose of CPD might be to update teachers' knowledge – of legislation, curricula or resources for example – the objective of many CPD initiatives has been to bring about some kind of 'change' in teachers themselves. The direction of any such intended change necessarily follows from a judgement by someone or some body about one kind of teaching practice being, in some sense, preferable to another. Such a judgement may or may not be evidence-based, and it might not be universally accepted that the proposed change will be for the better. Aspects of educational policy determined by the uninformed preference of a single government minister is not unknown in the UK (see e.g. Rowland, 1999, pp. 178-9).

In any case, change is a complex and unpredictable process. The director of one mathematics teacher development project in the 1980s wrote "Effective [teacher] change will take time and effort. There are no short cuts" (Ahmed, 1987, p. 36). More recently, the external evaluators of the National Numeracy Strategy (NNS) have concluded that bringing about change is "Hard Work – Intellectually and Emotionally" (Earl, Fullan, Leithwood, and Watson, 2000, p. 39). This conclusion is supported by a study in the USA (Clarke, 1997) of two lower-secondary teachers in the same school with similar professional support networks who were involved in a long-term and intensive programme of professional development. One teacher's practice changed significantly, whilst the other teacher's practice did not change. Clarke concluded that the teacher who did not change simply did not have the space in her life at that time to make substantial changes to her practice. Nolder and Tytherleigh (1990) found that the impact of the LEA-based intervention, even

working with hand-picked participants, was tempered by pragmatic constraints in these teachers' schools. They report that for many schools "the change effected was not as radical as was initially envisaged and was to some extent a compromise" (p. 20).

The influence of teachers' pre-existing beliefs and attitudes on their propensity to change is highlighted by Cooney (2001) in a review of five studies (one UK, four USA) in which "teachers' conceptions about mathematics and mathematics teaching strongly influence, if not dictate, their movement towards a reform-oriented teaching environment". In the first of these studies, Wilson and Goldenberg (1998) identified a reluctance to change too radically even in teachers who were well disposed to reform. Grant, Hiebert and Wearne (1998) studied 12 primary teachers who attempted to reform their mathematics teaching and found that those who had a more 'pluralistic' view of mathematics were more likely to show improvement. Schifter (1998) reported a case study of two participants in a seminar for teachers in which big ideas in mathematics were examined. These teachers came to a better understanding of the problematic nature of students' mathematical thinking as they reflected on the problematic nature of their own mathematical learning. In the fourth study reviewed by Cooney, Jaworski (1998) identified a reflective and cyclical process of teacher research that involved teachers questioning and examining their own teaching and "through which teachers' knowledge grew and was refined" (p. 26). Finally, Frykholm (1999) tracked 63 secondary teachers on a professional development programme, concluding that beliefs alone did not necessarily translate into corresponding classroom practice even when the two were in direct conflict. This echoes one of the findings of Desforges and Cockburn (1987) - the possibility of a mismatch between teachers' articulated beliefs about learning and their classroom practice.

There is also evidence that teachers' *perceptions* of their own change do not always match with their classroom practice (e.g. Spillane, 1999). Millett (1996) identified the particular problems in mathematics where teachers collectively lacked the confidence to discuss issues in depth and to take risks in changing practice. Askew *et al*. (1997b) found schools improving the effectiveness of their numeracy teaching given the availability of expertise, committed leadership, a substantial investment of time over several years and a cross fertilisation of ideas. Spillane (1999) also suggests that collaborative professional networks are important for teachers to make sense of the ideas promoted by reformers. Spillane reports that, of the teachers who thought they had changed with respect to the implementation of the 'reform' standards, very few actually had changed. These findings were replicated with respect to the implementation of the National Curriculum in an evaluation carried out at King's College, London (Johnson and Millett, 1996). See also Cohen (1990).

The transient nature of the school community is another factor that can disrupt the process of change. Instability can rapidly beset a small primary school due to individual teacher priorities and interests at particular times or external factors. For

example, in one focus of the King's College Leverhulme Numeracy Research Programme (Adhami, 2000), eight of the original 18 teachers left their schools at the end of the first year of a two-year year professional development programme. Notwithstanding evidence that long-term CPD is effective in bringing about teacher change (Askew *et al*., 1997), such an approach is particularly vulnerable to teacher mobility. Some attrition in this project appeared to be associated with high mobility in teaching generally, and London schools in particular; some of it due to the NNS; and some of it was particular to the group of teachers who were the subjects of the study. Similar levels of mobility were reported in a study of two three-year professional development programmes in Manchester (McNamara, 2002).

Smith (2000) found that almost all teachers on a long-term professional development initiative disliked the long-term nature of the programme. Postlethwaite and Haggarty (1998) note that there is very little literature about teacher change where teachers have not volunteered to take part; we know much more about teachers who *want* to change. Much of this research is rather project specific, and is reviewed below.

We simply note here that research on the development of mathematics teacher *educators* is difficult to locate. Being a 'teacher' of mathematics teachers involves skills and understandings in addition to those required to teach mathematics *per se* (Prestage and Perks, 1999a, 2000). Moreover, supporting the professional development of experienced teachers calls on approaches and, perhaps, attitudes different from those required in the initial training of novice teachers. The *Journal of Mathematics Teacher Education* (JMTE) includes articles that start to document aspects of teacher educators' learning related to their engagement in programmes for teachers of mathematics (e.g., Heaton and Mickelson, 2002; Remillard and Geist, 2002). Research on how teacher educators learn their profession, and how models of teacher professional development might extend to teacher educator development, is almost non-existent. Fragments can be found in the work of Halai (1998); Krainer (1999); Zaslavsky and Leikin (1999); Jaworski (1999a, 2000a); Prestage and Perks (2001); Tzur (2001). The issues also differ for 'career' mathematics educators based in universities and those chosen for that role in the context of particular curriculum development initiatives. The National Numeracy Strategy, for example, has a 'training' model of teacher development in which practitioners (Numeracy Consultants) 'deliver' a packaged training programme for the Strategy. The primary expertise of many such consultants has been gained as a generalist schoolteacher; their actual knowledge of mathematics and mathematics education may be modest.

## From Nuffield to Numeracy: Four Decades of CPD

Two major curriculum development projects in primary and secondary mathematics, Nuffield Primary Mathematics (1966-71) and SMP respectively, were both funded by charitable bodies. Each had its associated programme of teacher development to support curricular and pedagogical reforms. The first LEA teachers' centres, for example, were an outcome of the Nuffield project (Watson, 1976), meeting a need for locally-based venues to house resources and to host meetings of teachers. The project

took a principled decision to develop printed resources for teachers – the Nuffield Guides (e.g. Nuffield Foundation, 1967, 1973) – and not to produce textbooks for children. Following two HMI Reports (DES, 1978, 1979), the Cockcroft Report (DES, 1982) and an HMI discussion paper (DES, 1985), the government of the day gave unprecedented support for professional development in mathematics throughout the 1980s (Biggs, 1983; Pinner and Shuard, 1985; Pirie, 1987), with direct funding for higher education accredited programmes such as the Mathematical Association Diploma in Mathematical Education (Melrose, 1982, 1983). The 1980s also saw a number of government- funded CPD programmes linked to post-Cockcroft curriculum innovation projects. The Low Attainers in Mathematics Project (LAMP) (Ahmed, 1987) and its successor, the Raising Achievement in Mathematics Project (RAMP) (Ahmed and Williams, 1991), addressed investigative approaches to teaching secondary mathematics. In the primary phase, Primary Initiatives in Mathematics Education (PrIME) 1987-92 (Shuard *et al*., 1991; Rowland, 1988/1994) addressed a comprehensive range of Cockcroft implementation issues, notably the integrated use of hand-held technology – the Calculator-Aware Number Curriculum (CAN).

In response to the implementation needs of the first National Curriculum in 1989, 20-day courses were introduced by the DES in 1990 for primary mathematics (and science) teachers through the LEA Training Grant Scheme (LEATGS). The programme aimed to enhance subject knowledge, pedagogical and dissemination skills for the implementation of the new National Curriculum. 20-day course opportunities, which spanned most of the 1990s, have in turn given way to another government priority, the introduction of the NNS in 1999. The implementation of the Strategy, together with its associated cascade-training programme, has perhaps been the first example of synthesis of wholesale intervention and professional development in an unprecedented move towards a prescribed pedagogy for primary teachers, extended to lower secondary in 2001.

Running alongside these latter two government initiatives over the last decade a raft of modular masters, and other higher degrees, have become an established route for teachers voluntarily undertaking CPD. Many of these award-bearing courses have included ‘teacher research’ as one assessed element, typically towards the production of a dissertation of some kind. A more recent innovation is the plethora of government initiatives which, broadly speaking, go under the banner of teacher–as-researcher in ways not traditionally associated with award-bearing courses. They include the teacher research grants programme funded by the Teacher Training Agency (TTA); school-based research consortia funded by the TTA and the Centre for British Teachers (CfBT); the DfEE ‘training school’ initiative that claims research as one of its four main strands; and, most recently, the DfEE Best Practice Scholarships. Many of this latter group of initiatives are lodged within the rhetoric of evidence-based practice, a notion derived from a medical/nursing model of professional development.

A number of curriculum reform and professional development initiatives from the 1960s (Nuffield Primary Mathematics) to the present time (the NNS) have been directed at primary school teachers. As specialists in one or two subjects, yet typically required to teach across the whole curriculum, they face particular challenges with regard to professional development needs. Watson (1976) noted the willingness of primary teachers to learn and adopt new approaches. "Where their secondary colleagues have seemed at times unable or unwilling to recognise their need for updating, primary teachers have come forward willingly with enthusiasm and refreshing openness to new ideas" (p. 67).

This brief historical survey, and the sections that follow, review overtly mathematical CPD initiatives and opportunities. It is not clear to what extent it is useful to look at generic research on professional development to inform research and practice in mathematics teacher development specifically, as the knowledge base is extremely limited (e.g. Brown and Borko, 1992; Clarke, 1994; Darling-Hammond, 1998).

## Models of Development in Mathematics Teaching

### *The Mathematical Association Diploma*

The Mathematical Association entered the field of In-service qualifications for teachers in the late 1950s, in response to a report of the Association of Teachers in Colleges and Departments of Education (ATCDE) which found that only a quarter of mathematics teachers had a 'good honours' degree in Mathematics (Price, 1994). Although the first MA Diplomas concerned themselves entirely with improving mathematics subject matter knowledge, a new Diploma in Mathematical Education devised to meet the needs of teachers of children between the ages of 5 and 13 started in the late 1970s and ran throughout the 1980's as a two-year part time course at some 50 centres nationally. The course had four assessed components: mathematics, mathematical education, mathematical investigation and special 'Child Study' (Price, 1994). The 200 hours of tuition was initially available only in evenings, weekends and half-terms. By the mid 1980's Grants for Educational Support and Training (GEST) funding had became available for some day release. From 1985-90, the one-year Cambridge-based course, for example, enabled release from school for one day per week in the Autumn and Summer terms, and for five weeks full-time in the first half of the Spring term.

Most MA Diploma participants had responsibility for the subject in their schools, many being heads or deputies. An evaluation of the early years of the new Diploma (Melrose, 1982, 1983) reported that 60% of participants found both the mathematics and mathematics education components very or fairly helpful, compared with only 45% for the special study and 35% for the investigation component. 40% of students reported the course to be fairly effective, and 26% limited in its effect upon the organisation and leadership of their school. These rather depressing findings may reflect a radical initiative in its infancy; unfortunately there was no replication of the evaluation. By the end of the 1980s over 6000 teachers had embarked upon the

course. Many of them subsequently went into LEA advisory work. Arguably, the investment made in that period is now bankrupt, in the absence of any comparable opportunity for sustained CPD in primary mathematics.

The 1980s also saw the introduction of MA Low Attainers Diploma and a Diploma for Heads of Department, respectively attracting some 800 and 200 participants. Lack of funding for such substantial courses, coupled with the pressures arising from the introduction of the National Curriculum in 1989, led to the disappearance of the Diplomas early in the 1990s.

*LAMP*

One DES/LEA-funded post-Cockcroft curriculum development project, LAMP (1983-1986), was related to low attaining secondary pupils. Both LAMP and RAMP (see later) were based on the principle that " … improvements and change can only be sustained if teachers in the classroom believe in and support the developments taking place. Impositions from above are therefore unlikely to work". (Ahmed, 1987, pp. 81-2). 12 teacher researchers and a network of 200 teachers became involved, each being given some release from the classroom.

The aims of LAMP included encouraging teachers to change their attitudes to low attainers; using problem solving and investigatory mathematics; exploring informal approaches to assessment; changing students' perceptions of mathematics and enhancing their confidence and independence. The achievements of the project included enhanced pupil attainment, with increased numbers taking A level; teachers feeling greater satisfaction; greater collaboration and cohesion in departments; and teachers becoming more discerning about in-service provision and resources, enhanced involvement of parents. (Ahmed, 1987)

The project report includes recommendations for professional development in mathematics, including the need for classroom teachers to be more involved in their own professional development, and to be given sufficient time to pool experience and expertise with colleagues.

*RAMP*

LAMP led on to another three-year curriculum development project, jointly funded by the DES and 34 LEAs. RAMP (1986-1989) set out to demonstrate how pupils of all abilities and aptitudes were able to achieve better test and examination results, along with increased motivation, confidence and interest through 'using and applying mathematics'. 70 teacher researchers were released for one day per week and RAMP the in-service programme affected some 8000 teachers. The project report (Ahmed and Williams, 1991) concludes that teaching facts and skills in isolation from conceptual structures and general strategies can undermine pupils' confidence and competence. The report recommended that schools and LEAs should incorporate teacher development policies as an essential part of any curriculum planning; that long term research and development work in mathematics education should be

centrally co-ordinated and involve participation by practising teachers; and that a central body should be responsible for monitoring and quality control of the professional development of teachers.

As is often the case, one has to turn to an independent evaluation (Selinger, 1987) for a more critical perspective on RAMP and LAMP. In a generally sympathetic account, Selinger noted that teachers involved in these projects seemed to have changed their attitudes towards mathematics teaching, but questioned whether these teachers' classroom practices had actually changed. In a case study of two teacher-researchers, Selinger found that their beliefs and attitudes about mathematics teaching had changed, but that they had not sufficiently taken into account the reactions of their pupils. She also found that the teacher-researchers' attitudes about 'effective' INSET had changed; they believed that change takes time and requires the active involvement of participants. With reference to Selinger's research (*op. cit.*), Ruthven (1999) points out that the significant focus of both projects on reflective practice poses an epistemological problem: the developing shared understandings of an in-group of participating teachers gave rise to a tendency to discount the alternative perspectives of pupils and of teachers not directly involved in the projects.

*PriME*

The Cockcroft Report had pointed to the need for developmental work in mathematics teaching which would take into account the technological and social changes of the previous two decades, and draw on the resource of research and development which had become available. The PrIME project (Primary Initiatives in Mathematics Education) aimed "to help [primary] teachers to carry out the recommendations of the Cockcroft Report, taking account of the impact of the new technology" (SCDC, 1986, p. 1), with a comprehensive agenda including language in mathematical thought and learning, mathematical investigations and problem solving, the role of practical work, teaching styles, classroom organisation, gender, culture and the role of parents. PrIME is now best remembered for CAN (Calculator-Aware Number Curriculum), the component associated with developing the use of calculators.

CAN supported a number of primary schools in which each child had free access to a calculator, with integrated use of calculators in classroom mathematics activity. One study of the early stages of CAN in one LEA (Rowland, 1988/1994) noted that 6 to 7-year-old children exposed to a calculator-aware curriculum tended to develop early familiarity with large numbers and efficient methods for mental calculation with numbers less than 100. This study also noted that involvement in CAN had caused some teachers to raise their expectation of what young children can do and understand in school mathematics. Teachers acknowledged that participation in CAN had been a strong influence on the investigative patterns of teaching and learning which were developing in their classrooms. However, the same study reported that the least able children (in mainstream classes) seemed not to benefit from their access to calculators, and that procedural and apparatus-based approaches to teaching

mathematics were deeply ingrained. The CAN-appointed national evaluator concluded (Duffin, 1993) that the project had influenced the mathematics curriculum, supported the development of an investigative teaching style and empowered children. The PrIME project team's CAN report (Shuard *et al.*, 1991) is a vivid account of the experience of pupils and teachers involved in the project.

*Twenty-day courses*

These specially designated courses for primary mathematics (and science) teachers were established in 1990-91in a ten million pound DES programme funded through the LEA Training Grants Scheme (LEATGS). One of the requirements for LEATGS funding and course 'designation' (i.e. publication in an approved list) by the DES was LEA/HEI partnership, with teaching and assessment undertaken by one or the other, or jointly. At one extreme, the Open University produced materials (Mason *et al.*, 1990) intended to support courses wholly provided by LEA and school personnel.

The declared aim of 20-day programmes was to enhance the subject knowledge, pedagogical and dissemination skills of primary school teachers for the introduction of the National Curriculum. The first year of the planned three-year programme was evaluated by Harland and Kinder (1992) at NFER (see also DES, 1991a). This evaluation covered a number of course models, including those 'delivered' by the LEA alone. Patterns of attendance tended to be one day per week. Assessment of course participants was undertaken or overseen by the HEI, and many gave credit towards Advanced Diplomas (and even Masters degrees) for successful course completion.

The brief for the NFER evaluation reflected a new concern at the DES for 'effective' (in terms of the impact on whole-school practice), value-for money course provision by the partnerships. Related issues concerned (a) the extent to which HEI mathematics specialist personnel involved in INSET were sufficiently in touch with primary school pedagogy (b) desire for rigour in assessing learning outcomes for course participants (c) the concern of some LEAs to use LEATGS funding as one way to hold on to advisory staff in the face of growing devolution of central LEA budgets to schools. These factors underpinned a certain tension in partnerships: LEAs needed an HEI partner to attract the funding, but many were reluctant to buy their teaching and accreditation services (such as credits towards Diplomas).

The main findings to emerge from the evaluation included:

1. the opportunity to attend in school time and the quality of course tutoring made it a qualitative improvement on participants' previous experience of INSET;
2. participants reported greater confidence in their subject understanding and enhanced capacity to teach open-ended, practical and investigative activities;
3. 'interactive presentations' using short exposition and focused questioning within extended opportunities for experiential learning was seen by participants as a model of good practice for classroom teaching and dissemination;

4. the prime influence on teachers' capacity to disseminate was confidence in their knowledge and understanding of the subject and its processes. Consequently, the effectiveness of any generic dissemination training is questionable;
5. weak course-school links: the recruitment of advisory teachers as course tutors by HEIs was evident, but there was a low incidence of intended advisory teacher follow-up in schools;
6. little interest in the 'carrot' of HEI academic awards: only a minority of participants sought HEI accreditation and it was rarely a motivation to attend. The necessity to write and submit assignments as a condition of accreditation was a major disincentive;
7. there was little evidence of substantial impact on participants' schools during or immediately after course attendance. The report questions whether most primary schools can provide either the time or the *milieu* to ensure that benefits will extend significantly beyond the course participants.

In the middle-to-late 1990s, 20-day courses gave way to 10-day and even 5-day courses addressing restricted elements of the National Curriculum, and eventually to 3-day NNS training. There is clear evidence from Askew *et al.* (1997a) that distinctive professional advantages accrued to teachers who were able to participate in the more sustained, longer-term training course.

*The National Numeracy Strategy*

Following the National Numeracy Project (NNP, 1996-1999), the NNS was designed to enhance the management of the numeracy curriculum and the pedagogic practice of primary teachers in interpreting and delivering the Mathematics National Curriculum Programmes of Study. Initial indications are that NNP and NNS have been well received with positive impacts reported upon teacher attitudes and their practices. (OFSTED, 1998, 2000; McNamara and Corbin, 2001; Millett and Johnson, 2000). The central figure in the NNS training model is the subject co-ordinator who, after a 3-day training provided by the LEA numeracy team, is subsequently expected to 'cascade' a prescribed INSET programme to their colleagues in school. Additionally the co-ordinator was given time-release to work alongside colleagues in their classrooms, and to give 'demonstration' lessons.

An evaluation of the first year of the strategy (OFSTED , 2000) located weaknesses in teachers' subject knowledge, particularly "teaching of progression from mental to written methods; problem solving techniques; and fractions decimals and percentages" (p. 6). Their report also identified cascade training as a weak link in the strategy. Although it acknowledged that the distance learning materials provided enabled co-ordinators to start to help teachers learn the principles of the strategy it noted that:

> given that the existing weaknesses in the teaching of mathematics are largely concerned with teachers' subject knowledge, there is a strong case for extending the training programme, as a matter of urgency, to a great number of teachers. (*ibid*, p. 7)

In the 'whole school action on numeracy' focus of the King's College Leverhulme Research programme, reactions to LEA training and subsequent school-based INSET varied from positive to highly critical. Notwithstanding this, by the end of the year, consistently positive reactions to the *strategy* (not the training) were being expressed by co-ordinators and teachers in all schools (Millet and Johnson, 2000; Mcnamara and Corbin, 2001). Millet and Johnson conceptualise the "increasingly demanding" nature of the co-ordinator's role as incorporating such additional features as "agent of imposed change" and "monitor/evaluator of practice". Mathematics co-ordinators were indeed central to three of the six key constructs developed to characterise differences between schools in the implementation of the NNS: enthusiasm/lack of enthusiasm for the role of co-ordinator, clarity/obscurity of vision, and balance/imbalance between head teacher and co-ordinator. There was also a suggestion that a combination of circumstances, personal qualities, expertise, relationships and decision making in certain schools provided a stable foundation for taking change on board.

The first annual report from the external evaluators of the National Numeracy (and Literacy) Strategies also identifies the limitations of the cascade model to transform instructional capacity where both "content knowledge" and "classroom practices" are lacking. The evaluators identified that after the "modest but significant gains" in numeracy, further improvement would be dependent upon "teachers learning new skills" (Earl *et al*., 2000, p. 7) and observe that present improvement may be as a result of "motivation" and "concentration of existing capacities" (*ibid*, p. 36). However the evaluation generally maintains the optimistic view that the strategies (for virtually all the findings in the report are common to both strategies) represent "impressive reform levers that have considerable potential to establish an infrastructure for lasting change" (*ibid*, p. 4).

### *Evidence-based Practice*

Davies defines Evidence-Based Education as "a set of principles and practices for enhancing educational policy and practice .. [that] operates on two levels. The first is to utilise existing evidence from worldwide research and literature… The second is to establish sound evidence" (Davies, 1999, pp. 108-9). Evidence Based Practice (EBP) is still in an emergent phase in education and this is signalled, in part, by the fluidity of the discourse: 'research-based' (TTA, 1996); 'research and evidence-based' (TTA, 1999); 'evidence-based' (Davies 1999);'evidence-informed' (Sebba, 1999). Initially promoted through initiatives such as the TTA teacher research grants scheme (1996-1999) and the Research Based School Consortia (1997- 2001); although similar it was ideologically distanced from the existing 'teacher-as –researcher' and 'classroom action-research' movements begun in the preceding decades (Elliott, 1991). Alongside *engagement in* research it privileged *engagement with* knowledge from the

wider research and evidence base to promote a 'what works', 'outcome oriented' model of educational research, a response, some suggest, to the 'crisis of legitimation' in educational research (Pirrie, 2001).

Ironically, current moves towards EBP in education are ideologically rather than empirically driven in that EBP itself is not as yet evidence-based. Initiatives such as those of the TTA, supported by Hargreaves' (1996a) TTA lecture, which sought to make teaching a research-based profession were based on extrapolations from other professional areas, such as medicine, where such professional practices were in the process of being implemented (Jacobson, Edwards, Grainier and Butler, 1997; Evidence-based Medicine Working Group, 1992). Such approaches remain controversial even in medicine, however, and their transferability to other professional areas such as teaching is contentious (Woolf, Diguiseppi, Atkins and Kameror 1996; Hargreaves, 1996a, 1996b, 1997; Hammersley, 1997; Foster, 1999; Elliott, 2001; Norris, 1996).

Some crucial issues underpin current debate. Interpretations of EBP range on a spectrum that, at one extreme, includes advocates who present it as an 'empowerment' strategy, as a formal and specialised knowledge base that would enhance professional autonomy and status. At the other extreme of the spectrum opponents expose it as an 'entrapment', a technology of teaching that denies craft knowledge and reduces capacity for professional action. There is also little consensus about what counts as evidence, inter or intra the major educational stakeholders (Baumfield and McGrane, 2000; Threlfall, 2000; Elliott, 2000) and in addition methodological debates are hotly contested (Evans, 2000; Elliiott, 2001; Hammersley, 2001; Oakley, 2001). Moreover, it remains unclear (a) for whom the evidence is intended e.g. policy makers, practitioners, parents, educational researchers etc.; (b) in what context is the evidence to be employed e.g. schools, individual teachers etc.; and (c) for what purpose is the evidence to be used e.g. to inform policy, to develop theory, to open new pedagogic possibilities etc.

It remains to be seen what model of EPB, if any, is applicable to education. Does the medical model fit? (Evans and Benefield, 2001) Without clear answers to such questions, the DfES has moved apace to provide an educational equivalent to the medical Cochrane Collaboration, which prepares, maintains and disseminates systematic reviews of effective health care interventions. The DfES 'Evidence for Policy and Practice Information and Co-ordinating Centre' is commissioning systematic reviews of school-based education research in order to build resource databases and help practitioners and policy makers to locate and access relevant research. See McNamara and Cobin (2001) and McNamara (2002) for extended accounts of these developments and debates.

*The Japanese Model*

A 'TIMSS Resource Kit' issued by the USA Department for Education (OERI, 1997) includes a videotape and case study of teaching and professional development in

Japan, significantly entitled 'Learning from Each Other'. We include a brief account of the Japanese approach here, in this otherwise UK-focused chapter, because it points to collegial attitudes to and modes of teaching development that will be explored more fully in the next chapter of this monograph. Whilst it is not clear how representative the video is of practice in Japan as a whole, it has attracted a good deal of interest as a model of CPD that affirms teachers' professionalism in an incremental drive to become better at what they do. Japanese teachers undergo a rigorous and intensive induction into the profession. A reduced teaching commitment allows for 60 days of closely mentored teaching together with a further 30 days at the local teachers' centre. Facilitated by a light teaching load compared with the USA and UK, the Japanese focus upon professional development continues systematically throughout a teacher's careers. After the 5$^{th}$ and 10$^{th}$ year of teaching, for example, teachers must attend resource centres for seminars, classroom observation and study (Jones, 1997). Throughout their careers, Japanese teachers engage in a continuous process of lesson development, meeting in 'lesson study groups' once a week for about an hour, to plan, implement and refine lessons collaboratively (Stigler and Hiebert, 1997, 1999) Teachers in Japan have an impressive level of professional competence and demonstrate high levels of mathematics subject knowledge (Whitburn, 1995, 1996). As a rule, schools encourage a high degree of interaction between teachers, and are characterised by a collegial approach to professional development (Japan Society of Mathematical Education, 2000). A similar climate of professional development seems to exist in China, where elementary teachers typically meet for one hour each week in 'teaching research groups' for 'intensive study of teaching materials' (Ma, 1999, pp. 136-7). Ma suggests that this is a significant factor in Chinese teachers' attainment of 'profound understanding of fundamental mathematics'.

## Conclusion

Current government policies aimed at raising standards in primary schools have been experienced by many primary teachers in terms of imposition and initiative overload. CPD programmes can be seen as part of a perpetual readjustment in teaching styles, related to the evolution of learning theories and policy fashions (Brown, 1997). In the longer term, however, embedding policy has not always been understood in the terms in which it was presented (e.g. Millett, 1996), nor has it always been fully implemented before the next policy was devised. It would seem that funded evaluation of long-term CPD programmes has usefully, but sometimes exclusively, been directed at the initial stages of such initiatives (Melrose, 1982, 1983; Harling and Kinder, 1992), whereas it would seem reasonable to expect a degree of 'bedding in' so that a mid-term evaluation would be more indicative of the success of the programme. Nevertheless, a number of messages come across very clearly from the models outlined above.

First, the need to allocate sufficient quality time to CPD, and to involve teachers themselves in the management of their professional development. Askew *et al*.

(1997b) identified extended mathematics programmes such as 20-day courses as the most effective way of changing beliefs and practices so as to improve effectiveness in numeracy teaching. In reality, however, primary schools still retain an individualistic notion of development and one-day courses still predominate by virtue of time and cost constraints and perceived needs (Bottery and Wright, 1996).

Secondly, the cascade training so favoured by a number of these models is persistently identified as the 'weak link' in the process. Evaluation after evaluation carry similar messages to that relating to the 20-day courses: "It is questionable whether most primary schools, in particular, can provide either the time or the milieu to ensure such transmission. Evidence, however, suggests that a prime influence on teachers' capacity to disseminate is confidence in their knowledge and understanding of the subject and its processes. Hence the advisability of general dissemination training was questionable" (Harland and Kinder, 1992, p. 6).

Finally, deep 'whole school' change and dissemination of good practice must be firmly rooted in teacher experience and depend upon cellular growth of working groups - a 'diffusion' rather than 'dissemination' model, of which the Japanese lesson study groups might be one example. Many of the initiatives described in this chapter seemed to set out to *change teachers* rather than involving teachers in change. The external evaluation of the NNS (Earl *et al*. 2000, p. 10) concludes that good training and strong support from head teachers will not be sufficient for the further embedding of the strategy. "In addition, it will be essential to create strong professional learning communities at the school level".

In conclusion, we identify the following questions. Firstly, in what ways do the mathematics professional development needs of primary and secondary teachers differ? In what ways are they affected by differences in subject matter knowledge, curriculum, professional motivation and identity? Secondly, are the Japanese/Chinese practices of intensive examination of pedagogy within schools transferable to the UK context? If so, in what ways could they be facilitated and made most valuable?

# 5 DEVELOPING MATHEMATICS TEACHING THROUGH COMMUNITIES OF INQUIRY AND CRITICAL INTELLIGENCE

*This chapter takes up notions of exploration, investigation and inquiry which have played an important part in developing teaching in mathematics in the UK over several decades. It recognises theory and research relating to the use of investigational work and inquiry in mathematical learning in classrooms, and critiques its institutionalisation as a result of government reports and assessment practices. Several sections of the chapter trace theoretical perspectives relating to inquiry approaches in mathematical learning and teaching development, including constructivism, sociocultural theories, communities of practice and communities of inquiry. The notion of critical intelligence is introduced to characterise the critical stance taken by teachers in communities of inquiry, and to highlight a dialectic between individual and social forms of learning in mathematics and mathematics teaching. A penultimate section on teachers researching teaching suggests the importance of links between teachers and educators in sustaining inquiry into teaching. The chapter concludes with a critical perspective on the implementation of inquiry approaches in school classrooms*

## The Inquiry Movement and its Origins

Many of the initiatives and programmes discussed in Chapter 4 have been related to or informed by a movement in the UK that has taken place over a number of decades, becoming obvious in the 1960s and 70s, which might be called loosely the 'inquiry' movement. It includes approaches to teaching mathematics involving exploratory or investigational work in mathematics lessons, and has developed from inquiry in exploring mathematics itself to inquiry into the associated teaching processes (Love, 1988; Jaworski, 1994). The latter can also be seen to stem from the teacher research movement in the UK, dating back to Stenhouse, as we shall discuss shortly.

Inquiry in mathematics itself has been a key component of courses such as the Mathematical Association Diploma and the 20 day courses. Teachers participating in these courses were introduced to mathematics through problem solving approaches in which they explored mathematical ideas in a range of topics and became familiar with the use of mathematical processes. Inquiry into mathematics teaching can be seen in the LAMP and RAMP projects, in PrIME, and currently in teacher research programmes such as those supported by TTA grants (See the TTA website on www.canteach.gov.uk/home.htm), and the more recent Best Practice Research Scholarships (www.teachernet.gov.uk/bprs). The LAMP and RAMP projects, as Selinger (1987) acknowledged, resulted in teachers' new ways of thinking about teaching, as a result of their collaborative explorations of curriculum ideas and approaches to teaching, albeit without corresponding change in the perspectives of their students and other teachers. *Collaborative* inquiry approaches are seen as particularly fruitful for teaching development in a number of parts of the world (for

example, in the Japanese model described above), as is being demonstrated through their debate in international conferences in Europe and more widely (e.g., Krainer, Goffree and Berger, 1999; Lin and Cooney, 2001, Wood, Scott-Nelson and Warfield, 2001).

The idea of *inquiry* as a root for practices in learning and teaching has long been important to educators in the UK It can be seen, for example, in Rowland's (1984), *The Enquiring Classroom*. It was fundamental to the Children's Philosophy movement involving a model for a classroom community of philosophical inquiry, with children engaged in the pursuit of truth/knowledge through inquiry, especially through dialogue. (e.g., Lipman, Sharp and Oscanyan, 1980; Splitter and Sharp, 1995). Inquiry (seeking to know through creative exploration) as opposed to discovery (trying to find out what *is*) in the UK developed from the work of Stenhouse in the Humanities Research Project and subsequent Ford Teaching Project (e.g., Elliott and Adelman, 1975). From such projects, over three decades, teachers across the curriculum, started to inquire into classroom processes and practices leading to the establishing of the action research movement in the UK (Stenhouse, 1984; McNiff, 1988; Whitehead, 1989; Elliot 1991; Pring, 2000). This included the formation of a teacher-research network, the *Collaborative Action Research Network* (CARN) and the establishment (in 1993) of the journal, *Educational Action Research*. For example, Atkinson (1994) writes about her action research into her role as mathematics coordinator in a large city first school, and highlights the tensions for a teacher in engaging in such research. Day (1999) analyses a position of 'teachers as inquirers' suggesting a commitment to inquiry is a significant factor in behaving as a professional.

The Cockcroft Report, the outcome of a government inquiry into the teaching of mathematics in schools, emphasised the importance of problem solving and investigational work in a range of teaching approaches that included also discussion between teacher and pupils, and pupils themselves, and appropriate practical work (DES, 1982, para. 243 ff). Publication of the Cockcroft Report, followed by a significant consultation paper from Her Majesty's Inspectorate (HMI) on mathematics teaching (DES, 1985), put the official seal of approval on inquiry approaches as a valued part of mathematics teaching in schools and led to more teachers using investigational work in classrooms. Some practical implications of using investigational work in classrooms will be discussed towards the end of this chapter.

The use of inquiry (or investigation, or exploration) in mathematics teaching was reflected particularly in the seminal book by Banwell, Saunders and Tahta (1972) which set the scene for inquiry in classrooms by offering a range of activities as starting points. Its interest for teachers may be seen in issues of the journal *Mathematics Teaching* where teachers wrote about their experiences of engaging students in investigational activities (see for example issues 71, 73, 75 in 1975 and 1976). Such writings may be seen as early manifestations of teachers' engagement in

classroom research into ways in which mathematical exploration can lead to students' conceptual learning of mathematics. Ruthven (2001) speaks of them as "a form of "popular research" involving careful observation of exemplary cases and systematic reflection on them" (p. 173). Inquiry approaches were institutionalized in 1988 with the inception of the GCSE examination (General Certificate of Secondary Education) at 16+ which included an element of *coursework*, often interpreted as *investigational* work, that was assessed separately from the formal part of the examination. One version of the GCSE examination, offered by the ATM (Association of Teacher of Mathematics), was examined entirely by coursework.

The origins of investigational work in mathematics classrooms in the UK can be seen as part of an international movement in mathematics learning and teaching in the 1970s and 1980s that promoted conjecturing classrooms and problem-solving environments in the learning of mathematics. Based on the work of George Polya (e.g. 1945) in the USA, leading proponents were, in the USA, Alan Schoenfeld (e.g. 1985) and, in the UK, John Mason (e.g. Mason *et al*, 1982). Emphasis was largely on processes and heuristics of mathematical problem solving (see also, Burton, 1984). Much of the early developmental work in the UK was only loosely research-based as we have seen above; however, it has been a forerunner of research in the USA where researchers have devised teaching experiments to create and investigate 'inquiry classrooms' (e.g., Davis, Maher and Noddings, 1990) and in the UK where investigative approaches have been studied and characterised (Jaworski, 1994). In both of these cases research was based in a *constructivist* philosophy of knowledge and learning, acknowledging overtly that such theoretical positions are significant in studies of learning and teaching in classrooms. This takes us into some of the literature on theories associated with investigative or inquiry approaches to learning and teaching.

## Knowledge Growth Through Inquiry

Collins (1988) speaks of 'Inquiry Teaching' as engaging the student in "using knowledge, so that it does not become 'inert' knowledge like much of the wisdom received from books and lectures". Such ideas date back to Emanuel Kant and further to Greek philosophers such as Aristotle; a little more recently John Dewey wrote

> … no such thing as imposition of truth from without, is possible. All depends upon the activity which the mind itself undergoes in responding to what is presented from without (Dewey, 1902/1990 p. 209)

The notion of inquiry relates particularly to perspectives in mathematics teacher education dealing with cognition and 'construction' of knowledge. Constructivism, rooted in the work of Piaget (e.g., Piaget, 1950) might be seen as describing cognitive activity in which individuals through processes of accommodation and reflective abstraction develop and modify mental schemata as their representations of knowledge. See for example, Steffe, (1977); Glasersfeld (1982, 1990); Cobb, (1988); Confrey, (1995); Goldin, (1998). Inquiry, or investigative methods in mathematics

teaching are seen to fit with a constructivist view of knowledge and learning as they offer challenges to stimulate mathematical thinking and create opportunities for critical reflection on mathematical understanding (Cobb, Wood and Yackel, 1990; Jaworski, 1994). Through interactions, physical and social, in the world around them, causing challenge to and reorganisation of existing mental formulations, learners are seen to develop *conceptual*, *relational* and *principled* understandings of mathematics (Skemp, 1976, Brown, 1979; Edwards and Mercer, 1987). In other words, learners go beyond the use and application of algorithms and rules to develop an understanding of *general* relationships in mathematics and to deal with problematic aspects of *abstraction* and *formalism* which are central to mathematics (Nardi, 1996).

Educators and researchers working with teachers have developed constructivist models to explain and guide developments in teachers' thinking (e.g., Carpenter, Fennema, Peterson and Carey, 1988; Cobb *et al.*, 1990; Schifter and Fosnot, 1993; Richardson, 1997; Jaworski, Wood and Dawson, 1999). Theories suggests that teachers will develop *conceptual*, *relational* and *principled* understandings of teaching that enable mathematical development in classrooms. Such models need careful scrutiny and critique as we shall address in Chapter 6.

Researchers and theorists, acknowledging the importance of communication and interaction and their resulting discourses to growth of knowledge in teaching have appended the word *social* to *constructivist*. So called 'social-constructivism', emphasises the importance of language and discourse in communicative environments through which learning occurs. Discussion of, and inquiry and investigation into classroom practices are seen to underpin knowledge growth in teaching and in teacher education (e.g., Simon, 1997; Lampert, 1998; Cobb and Bowers, 1999; Wood, 1999).

Although these theoretical positions recognise the importance of social interactions in promoting growth of individual knowledge, they focus largely on construction of knowledge by the individual learner. Challenges to these views have come from researchers and theorists working in Vygotskian traditions who locate learning centrally in the sociocultural environment in which the learner is situated. Individual learning is regarded as derivative of social learning (Vygotsky, 1962, 1978). Learning is seen as a process of social enculturation with the individual internalising knowledge from interactions in the social world (Bartolini-Bussi, 1994; Lerman, 1996). It is believed that such social location of learning, teaching and teacher education avoids our ignoring of pathological social circumstances that result in learners' alienation from schooling (Lerman, 2000). There is, however, vigorous defence of constructivist perspectives on knowledge and learning that challenges notions of internalisation and enculturation (e.g., Steffe and Thompson, 2000). *Intersubjectivity*, ways in which individuals and groups understand each other and construct reality, is seen as a key concept relating individual and social perspectives, although its roots are treated differently in the different camps.

A recent special issue of *Educational Studies in Mathematics* was devoted explicitly

to contrasting individual and social perspectives (Kieran, Forman and Sfard, 2001). Authors in this volume take *discourse* and *communication* as the basis of their analysis of classroom situations involving mathematical learning where "the focus is on the change generated by interpersonal interactions" which "results in a picture which is more complex and closer to life than in the traditional cognitivist studies" (p. 7). Such perspectives emphasis a concept of 'community' and it is through community that we propose some possible rationalisation with notions of inquiry.

**Communities of Practice and Situated Cognition**

There has been a growing interest in the importance of seeing learners less as individuals independently constructing knowledge, albeit within and supported by social interactions, and more as participants in and contributors to the social environment of the mathematics classroom, and beyond, in which learning takes place. In a classroom, learners can be seen to participate in a *community of practice*, the practice being that of mathematics learning and teaching. *Community of practice* is an extension of the socioculturalist position, mentioned above in which some regular integrated social purpose, and collection of norms and beliefs locate, or *situate*, knowledge and cognition *in* practice. The term 'communities of practice' has arisen with reference to the social communicative activity of groups based on social practices, such as those of tailoring, or alcoholics anonymous, (Lave, 1988; Lave and Wenger, 1991) and teaching (Lave, 1996; Adler, 1996, 1999; Wenger, 1998). Knowledge is seen to be in the practice and to grow through activity and interactions in the practice. For example, the novice tailor learns from absorption into the community of tailors, led by *master* tailors, and gradually reducing peripherality in the community through increasing participation in the community's practices. The process is seen as one of situativity, apprenticeship and enculturation (e.g., Brown, Collins and Duguid, 1989; Greeno, 1998a, b). Wenger (1998) emphasises the production of *identity* through participation in a community of practice. Learning is presented as a "process of becoming". Wenger states, "It is in that formation of identity that learning can become a source of meaningfulness and of personal and social energy" (p. 215). Linehan and McCarthy (2001) emphasise the need for community models to form a clearer conceptualisation of the 'often messy' relationships between individual and community especially where learning in classrooms is concerned (p. 129).

There are problems in extending ideas of community of practice to students' learning in classrooms, as it is not clear what exactly the practice is into which learners are being enculturated and of which teachers are 'masters'. Teaching and teacher education, on the other hand, *can* be conceptualised as involving communities of practice, with new teachers being drawn into the community through increasing participation modelled on existing well-developed practices (Adler, 1996). For any teacher, their employing school represents a well-established community of practice, distinct from, though overlapping with, the communities of practice which exist in other schools. In addition to the mediating effect of the individuals involved, the

organisational culture of the school will also affect teacher progress (Carré, 1993; Lave 1996). Nias, Southworth and Yeomans (1989) note the uniqueness of culture in each of their research schools and argue for induction processes which help new staff to become included and socialised into the culture of the school. Maynard (2000) confirms that student teachers, through engagement in staff room rituals and routines as well as through discussion with their mentors, are both able to learn from talk and to talk as legitimate members of the community. However, socialisation into existing cultures and practices can be seen also to perpetuate *ineffective* practices in teaching, with "some schools merely recycling their inadequacies" (Alexander, Ross and Woodhead, 1992, p. 53) and result in deficit views of teaching practice (Brown and McIntyre, 1993, Jaworski, 2000a).

So far we have not considered in what ways *inquiry* might form a part of such communities. A possibility is that including inquiry as a normative practice in a community might avoid the perpetuation of ineffective practices.

**Communities of Inquiry**

Alan Schoenfeld (1996) has described, vividly, a community of inquiry that developed through his mathematics education research group in a university environment. As researchers, members of the group were fundamentally engaged in inquiry. The power of the community could be seen in ways in which interactions within the group led to the growth of practices of questioning and critiquing perspectives in a mutually supportive fashion, so that relationships within the group were strengthened, a clearer understanding of inquiry, reflection and critique emerged, and knowledge of mathematics learning and teaching developed.

The term 'Communities of Inquiry' is used by Gordon Wells (1999) in a discussion of 'dialogic inquiry' rooted in the work of Vygotsky. Wells draws on notions of inquiry as "a willingness to wonder, to ask questions, and to seek to understand by collaborating with others in the attempt to make answers to them", and as a means to emphasise "the essential continuity of education (Dewey, 1938, 1956)". This continuity is shown through the use of inquiry by students in classrooms, teachers responsible for their education, and those who are responsible for teachers' initial preparation and continuing professional development (Wells, 1999, p. 122). Wells draws on interpretations of *community* by a number of authors including Brown and Campione (1994), Rogoff (1994) and Lave and Wenger (1991). However, he distinguishes communities of inquiry from communities of practice by highlighting the importance of "metaknowing through reflecting on what is being or has been contributed and on the tools and practices involved in the process" (p. 124). Wells' research focuses on teachers who are "attempting to develop such communities of inquiry and simultaneously making their attempts the objects of their own inquiries" (p. 124).

In the final chapter of a collection of papers called, *Understanding practice: Perspectives on activity and context*, one of the editors, Seth Chaiklin, writes as follows:

> Social science research has the potential to illuminate and clarify the practices we are studying as well as the possibility to be incorporated into the very practices being investigated. (Chaiklin, 1993, p. 394)

These words emphasise the nature of research, not only as a means to illuminate practice, but as a source of study in investigations of practice; the research itself being part of the practice under investigation. Both Wells and Chaiklin thus point to a community of inquiry as involving a dialectical relationship between a community of practice and its activities in inquiring into and developing practice.

Cochran Smith and Lytle (1999), referring to a conceptual framework for teachers' learning emanating from a three year study of the relationships of inquiry, knowledge and professional practice in urban communities in the U.S., introduce as a new construct 'inquiry as stance'. They use this construct to describe "the positions teachers and others who work together in inquiry communities take towards knowledge and its relationships to practice" (p. 288). Their usage and elaboration of this concept fits with our discussion above and below. These authors write:

> Teachers and student teachers who take an inquiry stance work within inquiry communities to generate local knowledge, envision and theorise their practice, and interpret and interrogate the theory and research of others. (p. 289)

Teachers taking an inquiry stance "[raise] questions about what counts as teaching and learning in classrooms" and "critique and seek to alter" systemic norms and relationships.

**Inquiry and Critical Reflection or Intelligence**

In a community of inquiry, the novice practitioner is drawn into the community through processes of observation, practice, questioning of practices, and inquiry into practice, as indicated in Schoenfeld's (1996) example. Wells (1999) emphasises the importance of *collaboration* between teachers and researchers in investigating ways of improving practice. At the root of such a model is the belief in a critical mode of reflective practice in which the roots of social engagement are challenged so that practices are continuously reconceptualised and developed for the benefit of participants. Cochran Smith and Lytle (1999, p. 289) suggest that "the work of inquiry communities is both social and political", aiming to bring about change in traditional ideas of knowledge and develop richer conceptions of practice. (See also the work of Carr and Kemmis, 1986, rooted in critical theory following the Habermas school). In the case of developing mathematics learning and teaching, reconceptualisation of knowledge and practice would be for the benefit of the learner of mathematics, enabling a better quality of mathematical learning to result.

Such a model contains an inherent dialectic (related to the one mentioned with

reference to Chaiklin above) in terms of its conceptualisation as an individual or social process. It is a social process in the sense that a participant is a member of a community (e.g. of teachers) with its own practices and dynamics of practice which go through social metamorphoses as inquiry takes place. It is an individual process in that individuals are encouraged to look critically at their own practices and to modify these through their own learning-in-practice. Developments within the community result from rationalisations, implicit and overt, between ongoing practices. Wenger (1998) speaks of "modes of belonging", including *engagement*, *imagination* and *alignment*. We engage with ideas through communicative practice, develop those ideas through exercising imagination and align ourselves, critically, "with respect to a broad and rich picture of the world" (p. 218). It seems possible to conceptualise inquiry in these terms.

The notion of Community of Inquiry might therefore be seen to draw together elements of (social) constructivism and elements of social practice theory: participants grow into and contribute to continual reconstitution of the community through critical reflection; inquiry is developed as one of the norms of practice within the community and individual identity develops through reflective inquiry. This combination can be seen as particularly relevant to the development of teaching through teachers inquiring into their own practices of teaching mathematics. To be sustained, inquiry must be overt to a considerable degree, and it is through individuals and groups making inquiry explicit that critical intelligence develops. In a context of 'mentoring' in ITE in mathematics, Jaworski and Watson (1994) contrast the "inner-mentor" of the critically inquiring individual, with processes of "co-mentoring" through which the community or group develops. Inner-mentor and co-mentoring are seen to be reflexive modes of a process of critical inquiry.

Research by Jaworski (1987, 1990) explored the development of teaching through teachers' use of classroom videotape as a reflective device to enhance teaching through both individual and group activity (Open University, 1990). At the same time a group working for the Mathematical Association abstracted a model from their own activity in developing teaching, known as *the anecdoting process* (Mathematical Association, 1991; Jaworski, 1991). This was based on teachers' stories or anecdotes in a variety of forms (including video recordings) used as a reflective device to promote the raising of issues and addressing of critical questions relating to teaching.

These initiatives, researched through teacher interviews and two conferences in Suffolk and Cumbria, showed evidence of teachers' critically reflective practice leading to teaching development of individuals and to thinking about teaching development within constituted groups. Critical intelligence resulted from an increasing awareness of issues and questions that practitioners had to face in the processes of developing teaching. The application of these processes to initial teacher education can be seen in the use of 'Episodes and Issues' sessions in which student-teachers bring 'episodes' from their classroom activity to share with colleagues in their university group as a stimulus for raising issues and critical questioning of

practices (Jaworski and Watson, 2001).

In the cases described above, communities of inquiry can be seen to have developed critical intelligence. The process of developing critical intelligence from which teaching develops can be seen as related to Schön's (1983, 1987) notion of *reflection-in-action* in which teachers recall issues discussed in critical reflection outside the classroom and act consciously in response to events in the classroom. Similarly, in a theory called the *discipline of noticing*, Mason (e.g., 2001) has suggested that critical reflection on past events, in collaboration with colleagues, can lead to *noticing in the moment* in practice, allowing the possibility for alternative decisions and actions. Examples of such noticing in classroom teaching and learning of mathematics may be found in Jaworski (1994).

Thus, theory suggests that teachers' reflective questioning outside the classroom, of practices *in* the classroom (reflecting on action), leads to a more overt in-classroom awareness of issues (reflecting in action) resulting in corresponding classroom action and, possibly, changes to practice. The research described here is small scale and localised, involving a few teachers in a few schools, which raises questions about the applicability of such processes or models for teaching development more widely (Eraut, 1994). Although anecdotal evidence exists to suggest that such situations occur, systematic research is needed to test these ideas and gain further insights to these processes.

**Teachers Researching Teaching**

Teachers' engagement in inquiry in teaching, involving both the use of inquiry approaches to classroom mathematical activities and inquiry into teaching itself, led to mathematics teachers engaging in small scale action research in classrooms. The *Association of Teacher of Mathematics* initiated a group of teacher researchers in the 1980s, leading to a publication entitled *Teacher is/as Researcher* (ATM, 1987). The play on words here reflected the question of what exactly was the role of the teacher engaging in research.

In mathematics education around the world, projects focusing on teaching development in mathematics encouraged models of critically reflective practice resulting in the development of communities of inquiry, of critical intelligence within these communities, and examples of teacher action research. A working group at the conferences of the *International Group for the Psychology of Mathematics Education* (PME) studied processes and practices in this work over a decade (Zack, Mousely and Breen, 1997). In most cases, those leading such projects found their own thinking and practices developing alongside the teachers and students with whom they worked. For example, Jaworski, studying investigative practices in secondary mathematics classrooms found that *naive* questions, from the researcher's perspective, were seen as *hard* questions by teachers and led to teachers critically reviewing their own theories of teaching. A reflexive relationship between researcher and teacher led to each critically examining their bases of conceptualising teaching

and its development (Jaworski, 1994). In Austria, Krainer and colleagues (e.g. Krainer, 1993) developed a university course in which teachers, alongside educators explored aspects of teaching practice. The resulting inquiry led to developments in practices at all levels. In New Zealand, Britt, Irwin and colleagues (e.g. Britt, *et al*., 1993) worked with teachers from intermediate and secondary schools to enhance teaching through inquiry approaches. Learning at all levels resulted from mutual inquiry. Again in the UK, a project in Oxford was funded to explore developments in teaching resulting from teachers undertaking overt inquiry into their own practice (Jaworski, 1998). Findings showed teachers accommodating to notions of teacher-research. As confidence in research approaches developed, teachers gained insights into classroom approaches and ways of developing them that had previously been no more than implicit in their work. More recent research in the UK by a school teacher and a teacher educator jointly, involving inquiry into learning and teaching practices, was conducted by Brown and Coles in a project which saw teacher and students as a *community of inquirers*, together inquiring into aspects of mathematics (e.g., Brown and Coles, 2000).

A consequence that became clear from many of the projects above, was that teacher-research was hard to sustain without support or stimulus from externally based colleagues, such as university researchers, or from experienced researchers within a school environment. Recent initiatives on the part of the TTA, and DfES (described in the chapter on CPD above) into teacher research premised on notions of developing evidence-based practice, have acknowledged the importance of the relationship between teachers and their university colleagues. Modes of funding are now based on formally agreed relationships between schools and HE Departments to support research in schools and the hoped-for evidence-based practice. This coincidence of the inquiry movement and the official rhetoric offers perhaps an opportunity to take further the somewhat idiosyncratic developments in the inquiry movement through national projects with official backing and associated funding. A breadth of vision here could lead to establishment of wider communities of inquiry in which all learners (students, teachers, educators and researchers) engage in collaborative inquiry at a variety of levels for the enhancement of school learning.

**Implications for the Use of Inquiry Approaches in Practice**

Chapter 5 has followed, largely, a theoretical pathway that has situated ideas about inquiry approaches to doing and teaching mathematics in theories of learning and development. This concluding section will look at some of the practical implications of implementing inquiry approaches.

There is considerable evidence, reported from both academic research and popular research, of UK teachers' use of inquiry approaches in mathematics teaching that involve and inspire students and promote mathematical thinking of a non-ritualistic kind (Jaworski, 1994; Ruthven, 2001). In the 1970s and early 80s, such approaches to teaching were promoted mainly by enthusiasts such as teacher educators or members of the *Association of Teachers of Mathematics* in the UK (Love, 1988). With support

from the Cockcroft Report (DES, 1982) and the later inclusion of coursework for examination at GCSE (in 1988) inquiry approaches and classroom investigational work became more widespread and institutionalised. Such activity was engendered to fit with examination requirements: evidence of exploration, conjecture, formulation and testing. However, teachers' approaches to teaching were often geared more to exposition and exploration than to investigative work. Lerman (1989) describes a teacher who "went round the classroom offering advice such as 'no, not that way, it won't lead anywhere, try this'". According to Lerman, "there had been no opportunity for the teacher to discuss or examine … how [an investigation] might differ from 'normal' mathematics" (p. 73). Jaworski (1994) reports a number of dilemmas for teachers in developing investigative approaches to mathematics teaching; for example, "the teachers' dilemma", to "inculcate knowledge, while apparently eliciting it" (p. 123; see also Edwards and Mercer, 1987, p. 126). Another problem with institutionalisation was that in some classrooms investigative work took on a ritualised form (Love, 1988; Morgan, 1998): activity seemed to be reduced to a formalistic procedure such as trying out a small number of cases, and spotting and representing a pattern. Hewitt (1992, p. 7) asked, "Is the diversity and richness of the mathematics curriculum being reduced to a series of spotting number patterns from tables?" Ruthven reports on the activity of student teachers required to engage in a professional exercise involving investigation, and their subsequent critical examination of the outcomes. One student teacher, particularly, felt that certain investigations were "so tightly structured as to lose their key qualities". She pointed toward "teacher exposition dressed up as practical work", "right and wrong answers, with no creativity possible in the solution", and "the problem of time constraints" (Ruthven, 2001, p. 177). Such reported experiences reflect many of the difficulties teachers face in beginning to use inquiry approaches in classrooms where the focus is on students' achievement according to narrowly conceived goals within an increasingly prescribed curriculum (DfEE, 199b, 2001) that is not geared towards investigational work.

Evidence from programmes such as LAMP and RAMP showed that inquiry and investigative approaches to mathematics teaching in classrooms, the design of such teaching and subsequent reflection on outcomes resulted in changed thinking and (sometimes) changed practice for the teachers who engaged in them (Selinger, 1987). The observable effects of this changed thinking for pupils and other teachers, to whom the project was disseminated, were less positive. We mentioned, in Chapter 4, Ruthven's reflections on this research: the suggestion that such activity leads to "in groups" and "out groups", in which the former discount the views of the latter. Thus, although this project can be seen as effective in teachers' changed thinking and practice, its outcomes for pupils and dissemination to other teachers seem not to be effective. Selinger wrote, "[The teacher researchers] now believe effective in-service strategies should be provided through active participation of the recipients" (Selinger, 1987, pp. 57-9; quoted in Ruthven, 1999, p. 206).

The above discussion points to two main issues related to the successful implementation of inquiry approaches; the first might be seen to be a consequence of the second:

- the issue of institutionalisation leading to a ritualised implementation that ignores students' creativity of thinking and associated conceptual understanding;
- the issue of teachers generally needing more opportunity to think through the philosophy of investigative approaches and to work on the dilemmas they raise for teaching.

Research has shown certain teachers, enthusiasts, or the 'in group' giving the time and thought (or provided, in projects like RAMP, with opportunity to think and explore) to developing such approaches and dealing with the dilemmas that arise. Disseminating the positive experiences of the 'in group' to enable other teachers to use inquiry methods in a non ritualised way has been shown, largely, to be unsuccessful. We suggest that forms of dissemination are problematic in that they use all too rarely methods of collaborative participation, relying instead on cascading information through written materials or oral exposition and demonstration. The work of the Cockcroft 'missionaries' was more successful, shown where advisory teachers worked closely, in the classroom and in workshops, with other teachers, encouraging participation and collaborative engagement with issues (e.g. Nolder and Tytherleigh, 1990).

Two exceptions to this somewhat uninspiring picture of implementation can be seen in initiatives which seem on the face of it to be assessment initiatives, but which had, or are currently having, significant influence on teachers and teaching with which they were or are involved, with, in one case, recognised gains in student achievement.

The first was the ATM GCSE (General Certificate of Secondary Education) by 100% coursework, mentioned above, which ran as a pilot project in six schools for five years from 1988 to 1992. The scheme for the assessment of pupils' work involved the domains of Communication, Implementation, Mathematical Knowledge, Interpretation, Mathematical Attitude and Autonomy. In reports from an evaluation, commissioned by the Association of Teachers of Mathematics (ATM), Love and Shiu (1991a, b) show that the assessment scheme resulted in teachers' development of evidence on which to base assessment, changed practice in the classroom related to finding evidence, and the development of curriculum related to conducting investigational mathematics. A study of the student performance by the examination board found a greatly increased proportion of girls gaining higher grades. Unfortunately, despite such positive indications from the pilot project, the government of the day decreed that coursework in GCSE would be reduced to 50% and later 20% of the overall assessment of GCSE, the rest being by examination, thereby ruling out the ATM 100% mode. Several accounts of the project by participants can be found in the literature (e.g. Ollerton and Hewitt, 1989; Watson,

1990; Ollerton, 1990; Sutcliffe, 1991; Askew and Haines, 1991).

The second project developed from an extensive survey of the research literature on classroom formative assessment (Black and Wiliam, 1998a) which resulted in a publication (Black and Wiliam, 1998b) setting out a strong case for effective teaching (in terms of pupil achievement) developing from teachers' engagement with formative assessment practices. The authors report on three areas in which teachers' practice was seen to develop, resulting in enhanced achievement for students, particularly those seen as low attainers: rich questioning, quality of feedback and the role of learners in their learning (Wiliam 1999a, b; 2000). These developments can be seen to embody significant aspects of communicative inquiry as discussed above. The project that developed from this earlier work was the *Kings-Medway-Oxfordshire Formative Assessment Project* based at King's College, University of London. The project involved an intervention with six schools in each of two Local Education Authorities. This consisted of a series of in-service sessions during which mathematics and science teachers (two of each, from each school) were introduced to principles of formative assessment, and visits to schools by members of the project staff. In the visits teaching was observed and teachers had the opportunity to discuss their ideas, and how they could be put into practice more effectively (Wiliam and Lee, 2001). The project was designed to build on the professionalism of teachers rather to impose a model of 'good formative assessment' on them. These reported aspects of the project suggest a communicative inquiry model in its implementation. The research design built into the project a (quantitative) measure of the effects of teachers' use of formative assessment practices, through a choice of suitable control groups for each teacher and the use of whatever assessment instruments the school used to measure pupils' attainment. Conservative estimates suggested an average gain equal to between one-quarter and one-half of a GCSE grade per student per subject, which would predict a significant rise in achievement across the whole school (*ibid*).

Finally, this chapter proposes that the collaborative engagement of students and teachers in inquiry approaches leads to situated and *principled* learning (Edwards and Mercer, 1987): learning that results from engagement with content, process and issues in a non-superficial (or non-ritualistic) way, dealing with the issues and dilemmas that arise. Those who teach, whether teachers teaching mathematics in classrooms, or teacher educators (including LEA advisors) teaching teachers, need opportunity to address the theory and philosophy that underpins inquiry approaches and struggle with the dilemmas that arise in practice. *Systemic* issues become obvious and give rise to questions such as: how can we rationalise an increasingly prescribed curriculum with encouragement of teacher groups to develop research-based, innovative approaches to classroom teaching? What forms of in-service activity encourage collaborative inquiry as part of the daily life of schools? How can government initiatives such as Best Practice Research Scholarships scheme be linked to wider school developments? The thrust of this chapter, and this final section in particular, suggests that such issues have to be addressed.

# 6 EPILOGUE

*The Epilogue 'looks over' the monograph as a whole to suggest key ideas, critical concepts and theory-practice links. In particular it examines concepts of 'warranted practice', addressing ways in which classroom practices can be grounded and evidenced; 'the pragmatic recontextualisation of theory', to rationalise apparent incommensurabilities in theoretical positions related to teaching, and learning to teach; and 'teacher knowledgeability' to address approaches to teacher learning that avoid deficit models of teachers' knowledge and practice. It goes on to consider methodologies in teaching and teacher education in a discussion of models of practice and their theoretical relatedness. In doing so it makes links to key questions raised in earlier chapters. A concluding section on 'inquiry, development and further research' argues for inquiry-based approaches that avoid deficit perspectives and address the problems and dilemmas of learning teaching; and collaborative research projects between teachers and educators to explore and enhance teaching development for students' more effective learning of mathematics.*

## Practice, Theory and Research

Our field is teaching development in mathematics and mathematics teacher education, principally in the UK. We have developed a strong sense of the wealth of practical knowledge rooted in experience that abounds, some of which is reflected in extensive and systematic research, other in small projects and the rest hardly at all in research terms. In a review of recent research in mathematics education, Askew and Wiliam (1995), looking only at research reported in refereed journals, commented how little of the extensive UK research was reported. This perhaps reflects what Cornu (2000), contrasting development in British and French systems of teacher education, refers to as Britain's "pragmatic and empirical culture". Whereas Britain seems to be "increasing still further its heavy emphasis on practical training in schools and de-emphasizing theory, France, which has an *academic* tradition in teacher training is also placing more importance on developing the practical side but within a framework that favours a dynamic interplay between theory and practice" (p. 199; our emphasis). Referring to Cornu, in their editorial introduction, Hoyles, Morgan and Woodhouse (1999) comment on the "refreshing" [for UK readers] acknowledgement by the French of the need to "utilize *collective competence* and take advantage of *research*" (p. 3; our emphasis). One inference is that, in the UK, we need to chart critically our collective competence and make our research more accessible and comprehensive in order to develop possible frameworks in which theory and practice can inter-relate dynamically.

In the same volume (Hoyles, Morgan and Woodhouse, 1999), Ruthven suggests a means to build on the pragmatic and empirical in a "warranted practice" that calls for "evidence over experience", "scrutiny over sentiment" and "argument over advocacy" (p. 212). The term *warranted* has two importance senses: "providing

reasoned grounds for the practice as intended" and "assuring that the practice as implemented does indeed realize its aims" (Ruthven, 1999, p. 210). Together with the suggested inference above, these aspirations offer important ideals for research in our field.

We are concerned here fundamentally with relationships between theory, research and practice. Ruthven refers to McIntyre's (1995) assertion that beginning (and practising) teachers should be encouraged to engage in 'practical theorising' about teaching: "the critical examination, development and experimental use of ideas from many sources, including both the elucidated practice of experienced teachers and also a diverse theoretical and research-based literature" (McIntyre, 1995, quoted in Ruthven, 1999, p. 212). In Chapter 4, we have pointed towards a number of UK programmes or initiatives where such practical theorising is evident: for example in the LAMP/RAMP projects in which practising teachers' collaborative participation in reflective activity in their teaching led to changed perspectives of such teaching; in the Oxford Internship Scheme in which reflective activity and small scale research is built into initial processes of developing teaching; and in small scale programmes of classroom research by practising teachers. However, often, such evidence of practical theorising relating to developments in teaching is seen not to have reciprocal effects in the experience and achievement of students learning mathematics. We can see in these cases evidence of 'reasoned grounds for the practice as intended' but little evidence of 'assuring that the practice as implemented does indeed realize its aims' (Ruthven, ibid).

Ruthven has talked about an "epistemology of professional judgment" (1999, p. 209) which carries with it both strengths and weaknesses. In Chapter 5, we have pointed towards a number of epistemological frameworks in which knowledge is conceived in a variety of terms, each focusing our attention onto particular modes, characteristics or dimensions of human learning and practice. Lerman (2001) speaks of an object of research on mathematics teaching and learning, from a sociocultural perspective, as "a particular moment in the zoom of a lens" (p. 87). The lens metaphor seems a useful one in addressing the layers of complexity here. Elsewhere, Lerman has spoken about a 'gap' between the inter-relation of theoretical and empirical fields in research and their practical manifestations in pedagogical practice (and ultimately official discourse) (reported in Jaworski, 2000a). What we notice all too rarely in the literature are the linkages between different positions on the zoom. Zooming in on pedagogical practice can present a very different orientation to that perceived when the focus is on the epistemological positions of those promoting such practice. It is complex and difficult to seek out these linkages: some researchers/theorisers point to essential incommensurabilities and others to some pragmatic recontextualisation of theory (Confrey, 1995; Jaworski, 2000b; Kieren, Forman and Sfard, 2001). Boaler (2000, p. 5) reminds us of the potential "schism" identified by Alan Schoenfeld in his presidential address to the American Educational Research Association, 1999 "between the 'fundamentally cognitive' and

'fundamentally social' studies of human thought and action" (p. 5). Dwelling on schism, or incommensurability seems to leave us within a realm of esoteric research and theory building that offers little to practitioners and policy makers.

Boaler (2000) introducing an edited book entitled '*Multiple* Perspectives on Mathematics Teaching and Learning' (our emphasis) follows Rogoff (1995) to suggest that "the realisation that learning is at any one time both individual and social requires that previously developed perspectives that give primacy to one or the other need to be adapted or combined" (Boaler, 2000, p. 6). A methodology of pragmatic recontextualisation seems necessary to reach towards such adaptation and explanation for unexplained factors in any theory. Thus pragmatic recontextualisation is a reflexive process that not only uses theory to suggest and promote modes of practice, but through interrogation of practice throws light on problematic aspects of theory. A pragmatic recontextualisation could involve refocusing on theoretical perspectives from the position of dilemmas of practice; for example, a project in Texas has been re-addressing constructivist theory in seeking to explain apparent links between social deprivation and poor achievement in mathematics (Confrey, 2000).

Such recontextualisation of theory is designed to explain the roots of dilemmas and enlighten practitioners in dealing with dilemmas. It points towards the development of methodologies for learning and teaching that are linked clearly to theoretical principles, and through which theories can be challenged. However, there are issues and dilemmas arising from practical implementation of theoretical ideas. For example, situated learning and a focus on language has been interpreted as requiring methods of group or project work; constructivist theory has been interpreted as requiring methods to promote cognitive conflict. While it is possible to make clear the theoretical links between these theories and methods, a danger is that the methods and associated practices assume regulatory mechanisms whose meanings and purposes are unclear, and sometimes damaging, to the learners they control (Boaler, 2000). For example, Halai (2001) shows how the well-intentioned use of documented strategies of *cooperative learning* in a mathematics lesson in Pakistan resulted in the alienation of one pupil whose fear of 'reporting-back' prevented his engagement with the mathematics of the activity.

This example highlights a number of kinds of issues related to the development of *warranted practice* (Ruthven, 1999) We see a problematic interface between Ruthven's principles of "providing reasoned grounds for the practice as intended" (in this case strategies of cooperative learning) and "assuring that the practice as implemented does indeed realize its aims". One set of issues is related to ways in which theories, such as that of cooperative learning, are interpreted and realised in mathematics classrooms; another set concerns teachers' understandings of pedagogic approaches, such as those *related* to cooperative learning, and their relation to principled learning of mathematics; yet a third set concerns ways in which *learning* is conceptualised as in 'cooperative learning' or in 'principled learning of mathematics'. These issues highlight the complexity of knowledge required of teachers in striving

for effective learning situations.

## Teachers' Knowledge and Knowledgeability

Chapter 2 has identified a range of elements of teachers' knowledge and has, in particular, problematized the relationship between content, or subject matter knowledge (in our case, *mathematical* knowledge) and pedagogical knowledge. The three questions raised at the end of Chapter 2 highlight the problems in teachers' development of mathematical knowledge, particularly syntactic knowledge; how university mathematics courses and teacher education courses do or can contribute to this development, and whether mathematical and developmental needs are different for primary and secondary teachers. The transformation of mathematical knowledge from 'learner knowledge' to 'teacher knowledge' (Prestage and Perks, 1999a) seems an important step in a teachers' growth of knowledge and in the development of teachers' *knowledgeability* (Lave, 1993; Adler, 2000).

Askew and Wiliam (1995), after reviewing research related to pupils' learning and related classroom approaches, state that "Many aspects of mathematics teaching are under-researched'. They point to the importance of focusing research on "the strategies which might be offered to teachers to affect their practice" (pp. 42-3). However, more importantly – as illustrated by the example from Halai above - seems to be how teachers develop knowledgeability of such strategies; that is principled understandings of the nature and purpose of such strategies that translates into classroom practice. Kowledgeability involves teachers in developing their ability to relate mathematical knowledge to such knowledge of strategies in ways that lead to effective mathematical learning of pupils in classrooms.

The concept of knowledgeability recognises overtly the deficit discourse we have mentioned in earlier chapters whereby through certain ways of expressing the knowledge that is expected of teachers (and indeed their students) we come to talk about what is lacking or missing and a concomitant necessity for remediation. Official rhetoric, seeking to improve educational outcomes, often contributes to the problem.

Adler (2000) introduces knowledgeability as follows:

> I use the concept of "knowledgeability" to capture and produce a dynamic and situated notion of learning, of coming to know about teaching. … . Knowledgeability contains within it a sense of being knowledge-able. As a positive statement, I believe, it provides a conceptual tool that could assist us to shift away from a deficit discourse in teacher professional development. (p. 33)

Adler's notion of situated learning accords with theories of communities of practice, Wenger's (1998) "process of becoming" and much current writing about teaching development that is rooted in social theories and referenced in Chapter 5. Teachers are seen to belong to multiple intersecting communities involving "social, political, economic, cultural and historical contexts" (Adler, 2000, p. 33).

Research needs to address how knowledgeability develops, from both situated and cognitivist perspectives, and what are the factors for teachers either as individuals or members of intersecting communities that contribute to knowledgeability. For example, how does a shift from knowing mathematics as a learner to knowing as a teacher develop? How is such a transformation related to development of pedagogy, and to mathematical learning in classrooms? These might be seen as questions relating to individual and cognitivist positions. On the other hand we might ask how communities of learners and of teachers of mathematics intersect. What are the nature and practices of such communities? How are teachers as learners of mathematics constituted and how do they relate to students as learners of mathematics?

Questions such of these have rhetorical force but little practical force. In their current form they are not researchable questions. To arrive at researchable questions a clear account of context, conditions and constraints is necessary – as in Confrey's research in Texas, or Halai's in Pakistan classrooms, or with teachers in the UK struggling to implement a highly specified numeracy curriculum. As we struggle with questions about teacher learning and its promotion we also face philosophical and epistemological issues in considering development of teaching knowledge. How to deal with dilemmas highlighted in Chapter 3, 4 and 5 is a part of this struggle. What kind(s) of research will address such questions and how can we arrive at questions which are researchable but not reductionist: i.e. which allow us to design specific lines of action within the complexities we recognise?

**The Theoretical Basis of Methodology**

Conclusions from Chapter 2 about the knowledge of teachers for teaching mathematics included the following:

a. formal qualifications are not reliable indicators of effective mathematics teaching (in the primary years);

b. 'secure' knowledge of mathematics (its modes of inquiry and the integrity or 'connectedness' of its content) is clearly associated with primary mathematics teaching judged to be effective;

c. there is little evidence to suggest that teachers' knowledge of mathematics develops as a consequence of teaching.

One inference, or proposition, from these statements combined is that formal qualifications do not ensure 'secure' knowledge of mathematics, and moreover, such secure knowledge of mathematics does not develop as a result of engagement in teaching mathematics. These are very interesting, although salutary, propositions. They offer further hypotheses or inferences leading to questions for future research, such as:

1. How can programmes leading to formal mathematical qualifications take account of findings regarding the nature of 'secure' mathematical knowledge, seen to be so valuable for teaching?

2. In what ways throughout training and subsequent practice in teaching, can teachers' knowledge of mathematics develop 'secure' forms?
3. How can we develop clearer theoretical and practical understandings of 'secure' mathematical knowledge and its relationship to pedagogy?

The idea of *secure* mathematical knowledge (or knowledge of mathematics) has currently a popular basis that is research related. It is also fundamentally *theory* rooted. Its exploration would be hollow without a clear epistemological analysis. Thus, the operationalisation of knowledge needs to be linked to the social epistemology of mathematics from which it is operationalised. From such considerations, a methodology of operationalisation can be sought; e.g. models for the development of secure forms of knowledge. Although use of such language makes the process seem technocratic, it is actually deeply social and developmental. It draws on all the theoretical positions outlined in Chapter 5, and challenges incommensurability or schism.

Experience and research has shown us that transmission models of knowledge transfer do not work; cascade models are one prime example. We might therefore look to models that are rooted in the theories outlined in Chapter 5. A danger as we have seen lies in trying to convert such theoretical perspectives into (simple) methodologies. Complexities of "multiple intersecting communities involving social, political, economic, cultural and historical contexts" (Adler, 2000, p. 33) have to be acknowledged and addressed with concomitant avoidance of pathologising particular groups of learners (e.g. with relation to race, gender or learning difficulty) (Adler and Lerman, 2001). The zoom of the research lens will inevitably focus attention depending on what is asked and how it is asked. The theoretical and practical implications of what is asked, and how, have to be acknowledged critically, as does, importantly, what is on the periphery of the zoom. This takes seriously Corbin's words relating to the incomplete nature of models of the knowledge required for teaching mathematics, quoted in Chapter 2: "part of their usefulness can be at their borders, in what they specifically exclude and include in particular instances of their use" (2000, p. 2).

Chapters 3 and 4 have made clear the proliferation of models in teacher education at both pre-service and in-service levels. The extent to which these models are research based, or theory explicit varies considerably. Although we can argue over their research-based nature, for example whether they rest on academically rigorous research or 'popular' research (as mentioned earlier), there is little doubt that few of them make explicit their theoretical/epistemological roots. These recognitions lead to yet more questions. Two questions expressed in Chapter 3 were:

- How can students [pre-service teachers] develop a capacity for working on their own professional development in a way that relates to their personal aspirations of what it is to be a teacher?
- How can we as educators model such activity as we balance the competing

demands of individual professional autonomy and collective state and student entitlement?

Asking 'how' leads to considerations of methodology. Our discussion here suggests that before we can address these methodological questions, and move towards models, we have to make explicit the theoretical perspectives that underpin the 'how' questions. For example, we can see individual or cognitivist elements in these questions (e.g., working on their own professional development; personal aspirations; individual autonomy) and sociocultural elements (e.g., pre-service teachers' [collective] professional development; collective entitlement). These terms need an epistemological analysis relating to mathematical and pedagogical knowledge and its social rooting in practices and relationships before research can be originated or models conceptualised. For example, the Japanese model, discussed briefly at the end of Chapter 4, seems to embody a number of these elements, such as individual or collective professional development; it also demonstrates teachers' clear focus on the mathematics to which teaching is related. We can analyse this model relating its particular characteristics to theoretical perspectives that explain and justify its approaches, while learning from the dilemmas that arise in its implementation. It is only by insisting on such epistemological rigour that is more than tokenistic *at the beginning* of the methodological process that models can be effectively scrutinised and evaluated. In all of this process, we need to keep in mind that our field lies within *mathematics* education, and our talk about models of professional practice such as 'partnership' and 'mentoring' must be firmly rooted in mathematical epistemology and well as social epistemology more generally.

**Inquiry, Development and Further Research: Conclusions**

In drawing this monograph to a close we shift from consideration of big ideas such as warranted practice, pragmatic recontextualisation of theory, and teacher knowledgeability, to a conclusion, a 'thesis' drawing on all our chapters and sections above. The teacher/teaching relationship was highlighted in the Prologue. While we might see ITT and CPD to be largely about teach*ers* and their development (both of which lead to teach*ing* development of sorts), DCI seems to be more explicitly about developing teach*ing*. Thus, while models of developing teachers can lead to a deficit discourse, we look to DCI models to offer an alternative approach to addressing the tensions, constraints and difficulties in teaching. So rather than looking at what is wrong with teaching, possibly in terms of a lack of evidence of students' 'effective' learning of mathematics, we explore together (where 'we' includes teachers and academic researchers) the issues of teaching and students' learning.

As we seek appropriate methodologies from which models can be formulated we need to make overt the theoretical/epistemological foundations on which we build. This itself involves inquiry (or meta-inquiry) and we have to take care not to engage in an endless epistemological spiral. There is considerable evidence, much of it very localised or small-scale, of genuine collaboration between teachers and educators in a spirit of inquiry, leading to development in positive directions (regarding students'

learning). The participants in such inquiry are likely to have very different experiences, knowledge, interests, strengths as well as roles and goals in the collaborative enterprise. In order to work productively for development, knowledge needs to be contributed according to its focuses and strengths. Thus, for example, the epistemological foundations of a model or project would most likely be the responsibility of those confident in such knowledge, whereas knowledge of the social setting, its complexity of inter-relationships might come from other partners. However, for effective outcomes, it seems to us that critical negotiation of all layers of knowledge, experience and decision-making are crucial. We do not underestimate the complexity and the problems that this poses at a variety of levels. However, over-simplification can lead not only to limited outcomes, but also to developmental disappointment and frustration.

To some extent we are arguing for projects of large scale and scope, and recognise those that are taking place currently in the second phase of the Economic and Social Research Council's (ESRC: www.esrc.ac.uk/) Teaching and Learning Research Programme (TLRP: www.tlrp.org/). None of the projects involved are overtly about mathematics, although mathematics learning and teaching are significant in at least three of them. Gaining large scale research funding at a national level is highly competitive, and it is likely that projects that could be hugely beneficial in the terms we have set out do not receive the necessary funding. Thus it is important not to undervalue the smaller studies that have been undertaken, and are being undertaken, but to recognise that these individually have less impact. Meta-analyses can help, and should be done more extensively. However, people initiating research at whatever level need to think clearly about potential developmental impact as well as esoteric contribution to knowledge. For example, those supervising doctoral research have a responsibility to their students and the wider community that the results of three years or more of intensive study should be known and used more widely than just the local research group.

While saying this, we recognise the movement towards evidence-based practice and the roles of classroom teachers engaging in research. Inevitably much of this work is extremely localised, and although funding providers encourage a variety of forms of dissemination, this itself needs scrutiny if it is to be an effective means of communication in terms of promoting teaching development through negotiative dialogue. The requirement for teacher research grants in the UK to be linked to a Higher Education Institution encourages such negotiative dialogue. As partners, the teachers, school and HEI academics have potential to achieve more than any one of them can independently. A basis of collaborative inquiry again offers epistemological coherence.

And so we arrive at and conclude with a statement of our thesis. We are arguing for inquiry-based research/development involving genuine collaboration and power sharing between teacher researchers and academic researchers in mathematics education with clearly defined goals directed at teaching development in order to

improve student learning, and with clearly articulated epistemological bases. All concerned have the important task of convincing funders and educational policy-makers that this approach has far-reaching potential to achieve the effective education of students in mathematics.

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# LIST OF ACRONYMS

| | |
|---|---|
| ATM | Association of Teacher of Mathematics |
| CATE | Council for the Accreditation of Teacher Education |
| CPD | Continuing Professional Development |
| DES | Department of Education and Science |
| DfE | Department for Education |
| DfEE | Department for Education and Employment |
| DfES | Department for Education and Skills |
| ESRC | Economic and Social Research Council |
| GCSE | General Certificate of Secondary Education |
| HEI | Higher Education Institution |
| HMI | Her Majesty's Inspector(ate) |
| ICT | Information and Communications Technology |
| ITE | Initial Teacher Education |
| ITT | Initial Teacher Training |
| LAMP | Low Attainers in Mathematics Project |
| LEA | Local Education Authority |
| MA | Mathematical Association |
| MOTE | Modes of Teacher Education |
| NNP | National Numeracy Project |
| NNS | National Numeracy Strategy |
| NQT | Newly Qualified Teacher |
| OFSTED | Office for Standards in Education |
| PCK | Pedagogical Content Knowledge |
| PGCE | Postgraduate Certificate in Education |
| PrIME | Primary Initiatives in Mathematics Education |
| QTS | Qualified Teacher Status |
| RAMP | Raising Achievement in Mathematics Project |
| SCDC | Schools Curriculum Development Council |
| SCITT | School-Centred Initial Teacher Training |
| SENCO | Special Educational Needs Co-ordinator |
| SMK | Subject Matter Knowledge |
| SMP | School Mathematics Projects |
| TIMSS | Third International Mathematics and Science Study |
| TLRP | Teaching and Learning Research Programme |
| TTA | Teacher Training Agency |